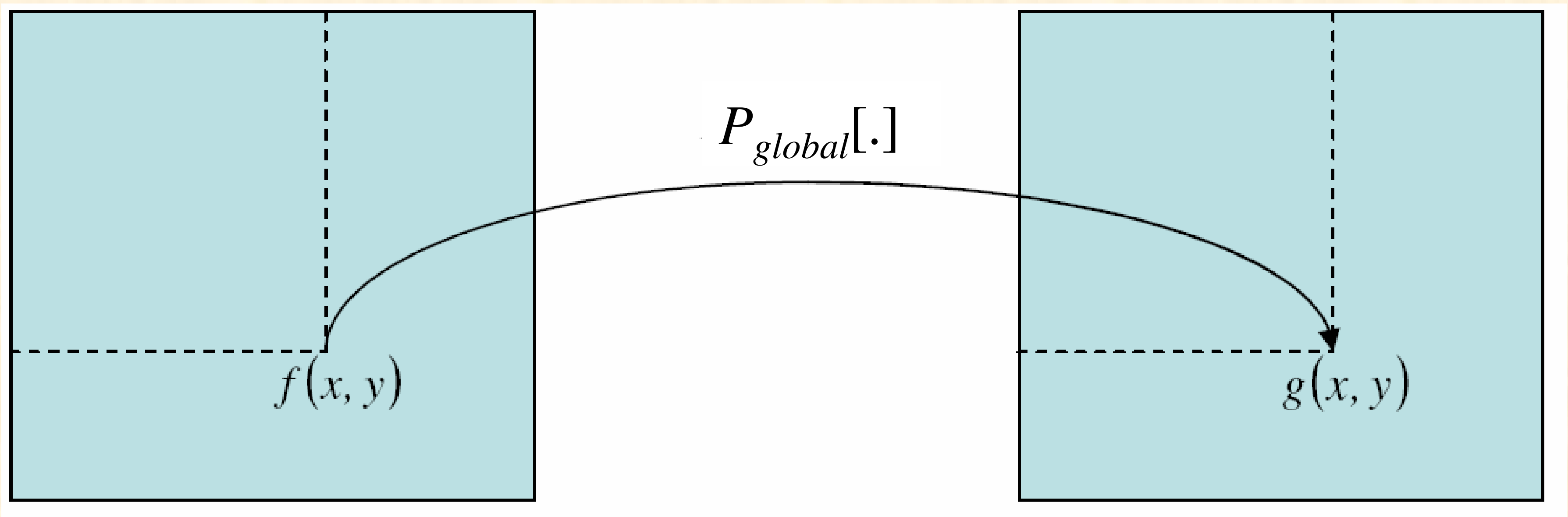
Image global operations



- **image global operations:**

  Aims:

  - zoom-in, zoom-out,
  - correction for image geometric distortions,
  - image translations and rotations.

Image geometric distortions

Visible geometric distortions due to camera optics

After correcting geometric distortions

Image geometric transformations

$f(x, y)$ → T[.] → $f'(x', y')$

$a<1$

$d>1$

Few parameters

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{aligned} x' &= ax \\ y' &= dy \end{aligned}$$

Affine transformations

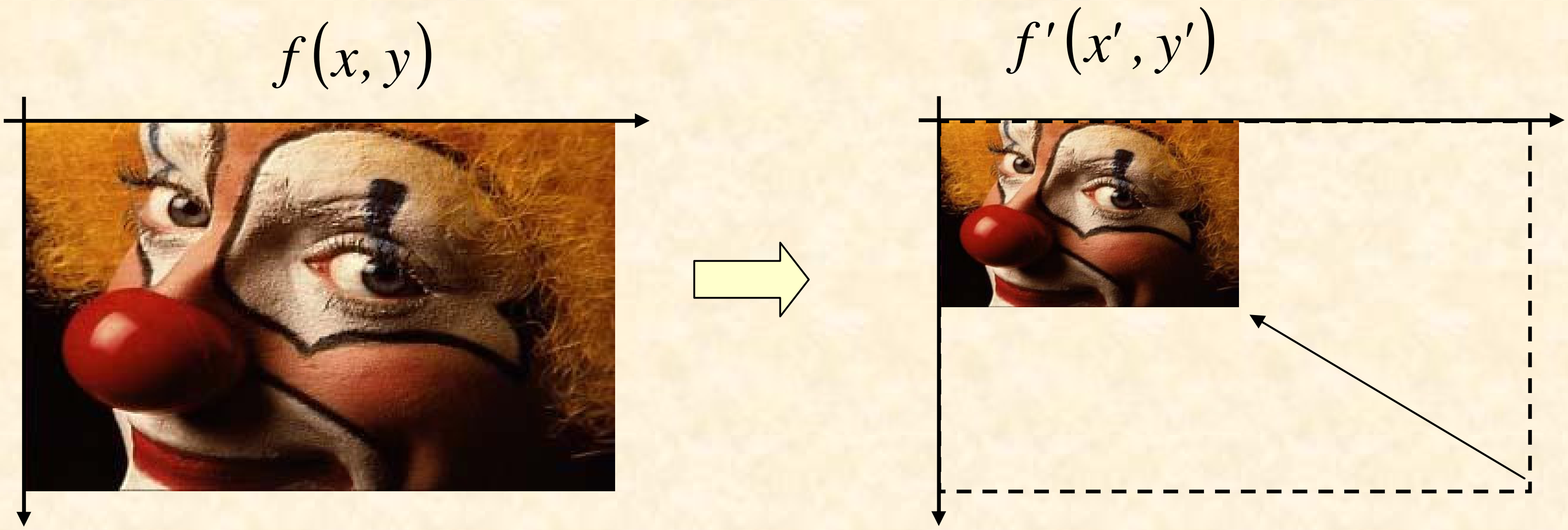
Rotation, scaling

translation

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix}$$

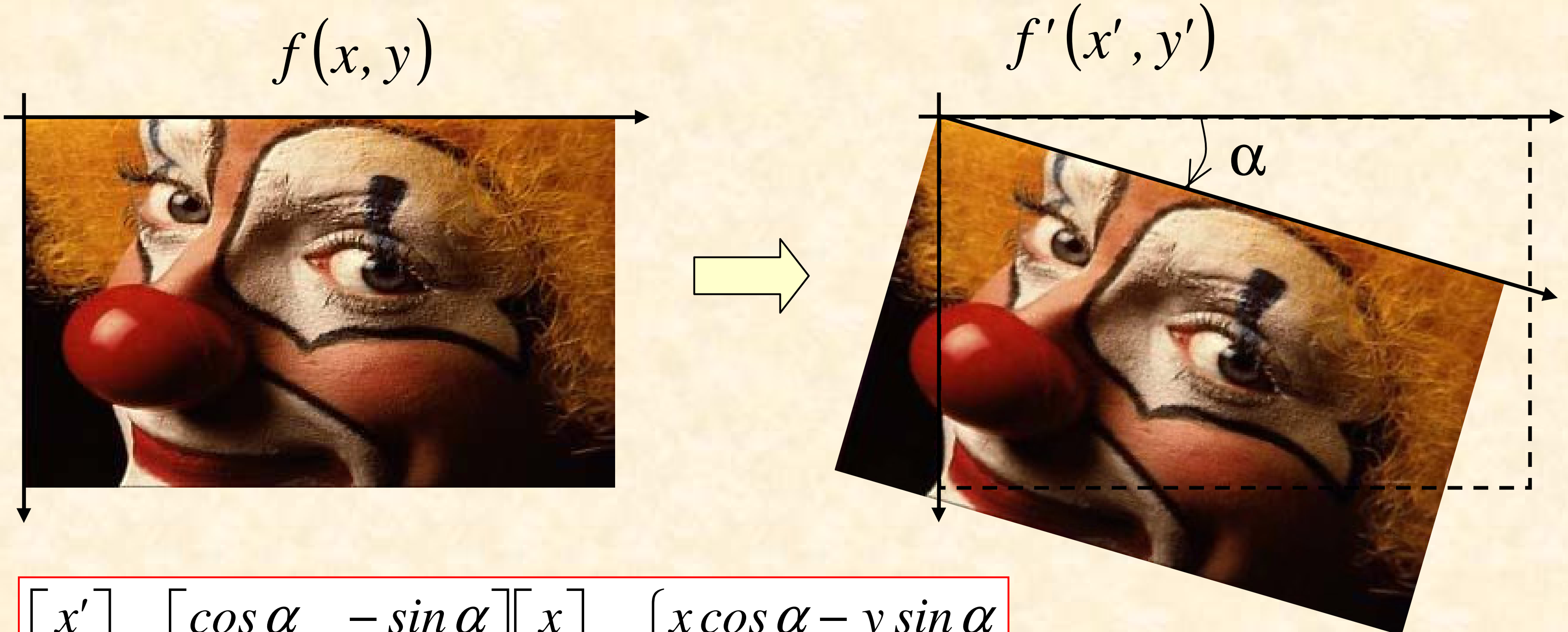
$$\begin{cases} P(x,y) = x' = ax + by + e \\ Q(x,y) = y' = cx + dy + f \end{cases}$$

Affine transformation example



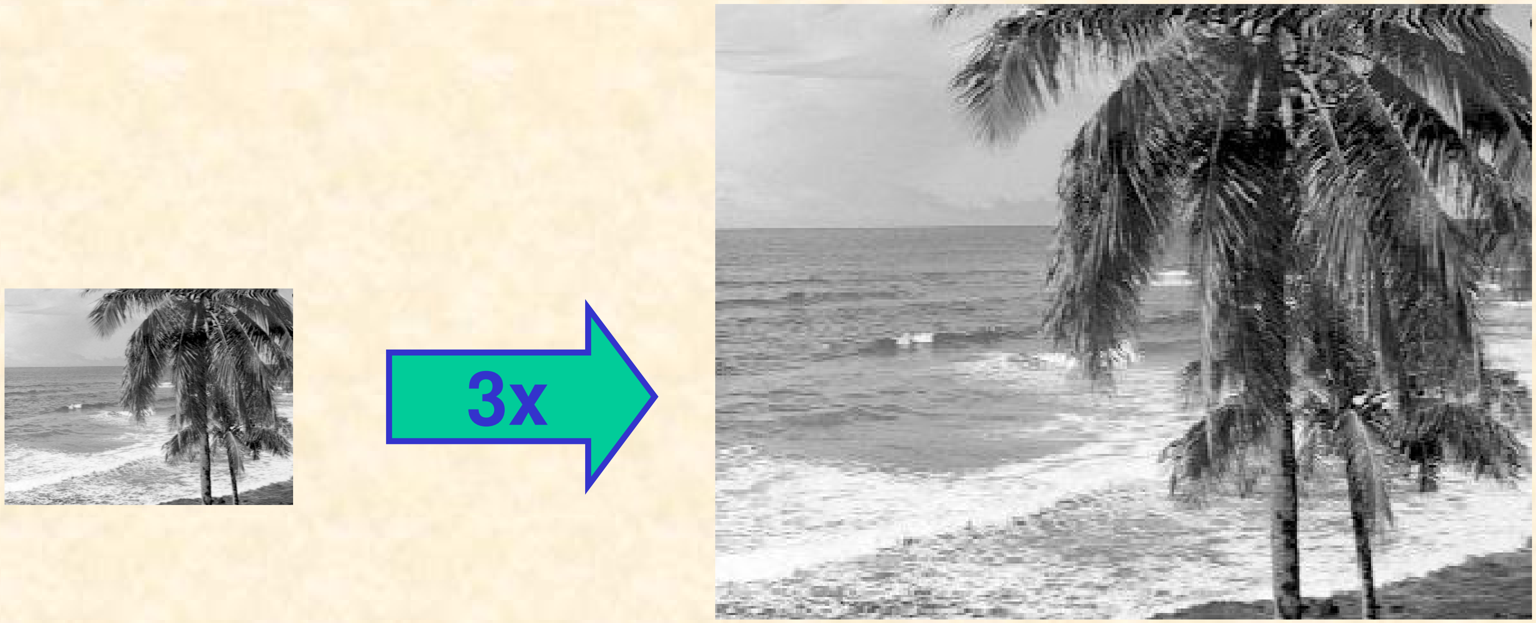
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

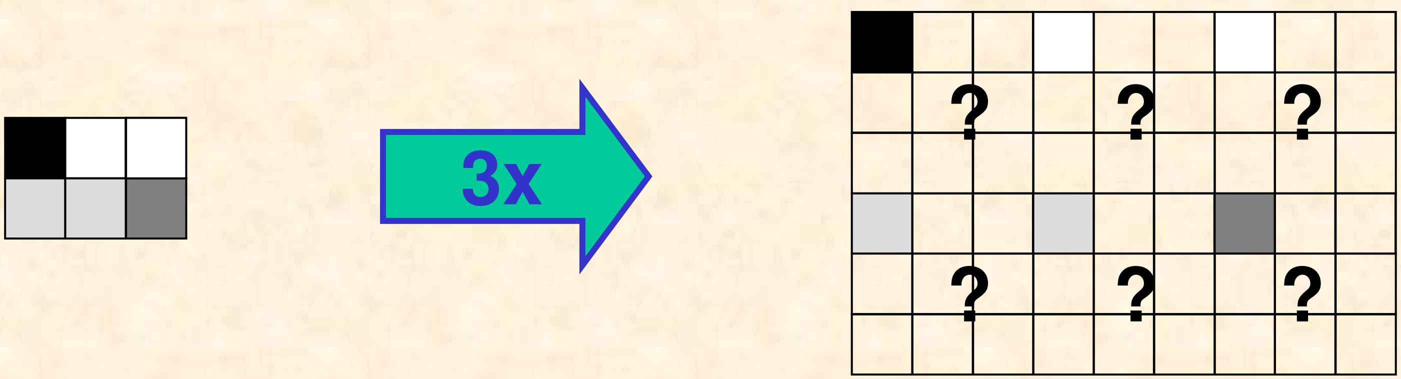
Image rotation



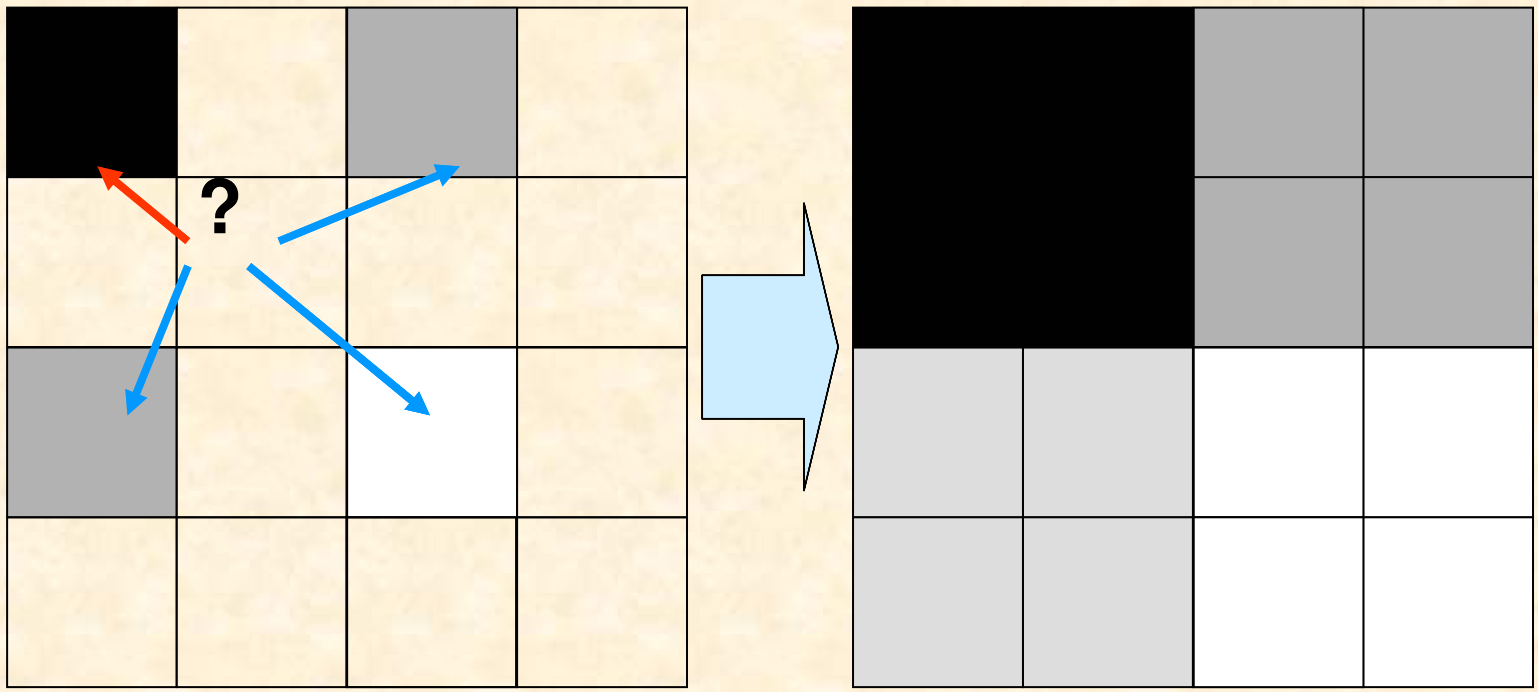
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{cases} x\cos\alpha - y\sin\alpha \\ x\sin\alpha + y\cos\alpha \end{cases}$$

Image enlargement



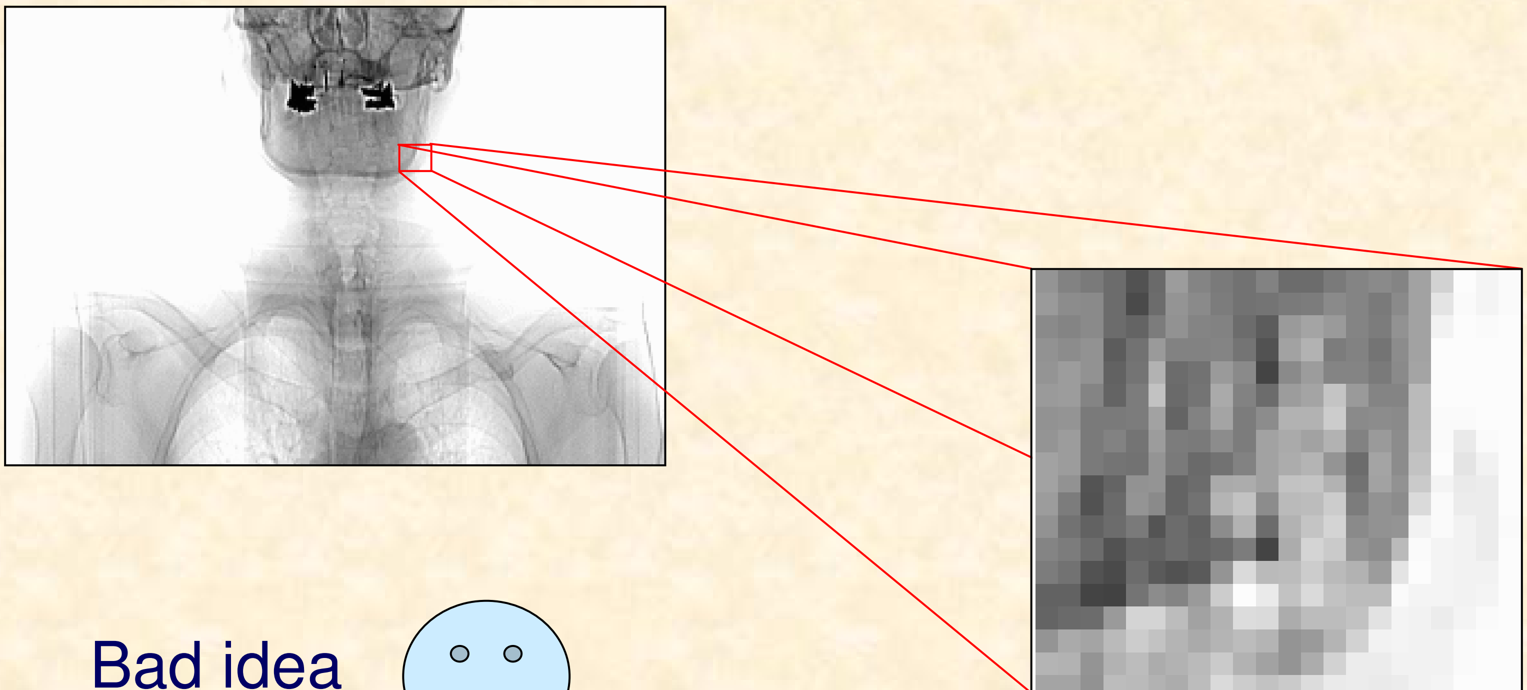


Nearest neighbour pixel replication



Zero-order interpolation

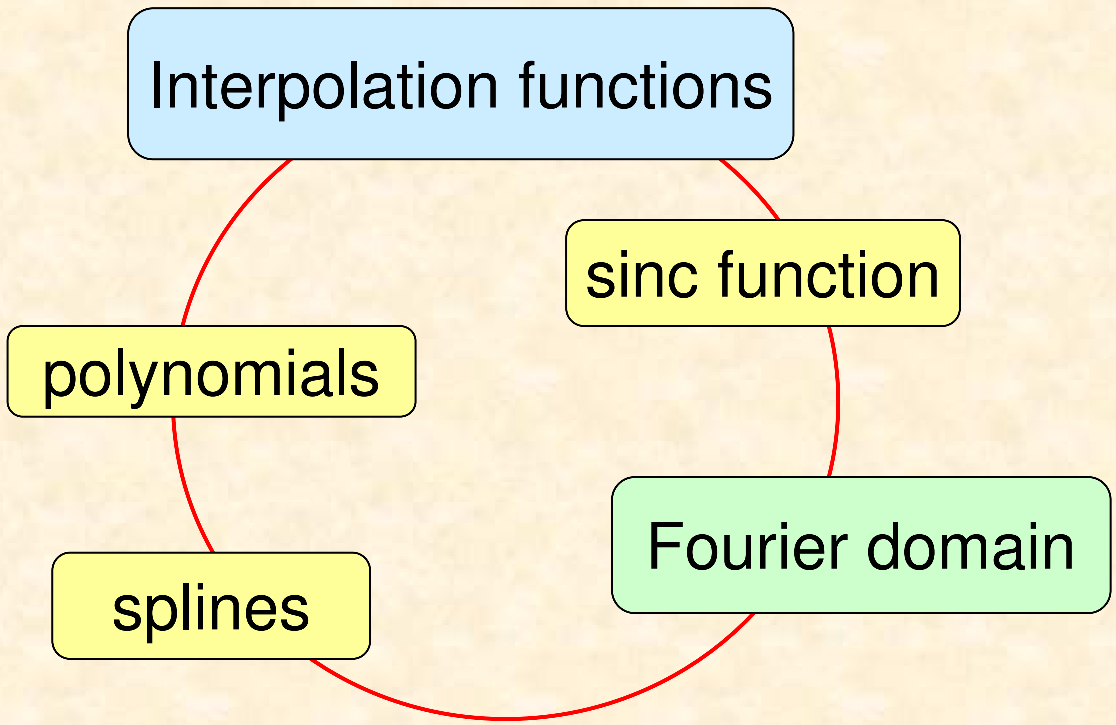
Pixel replication - example



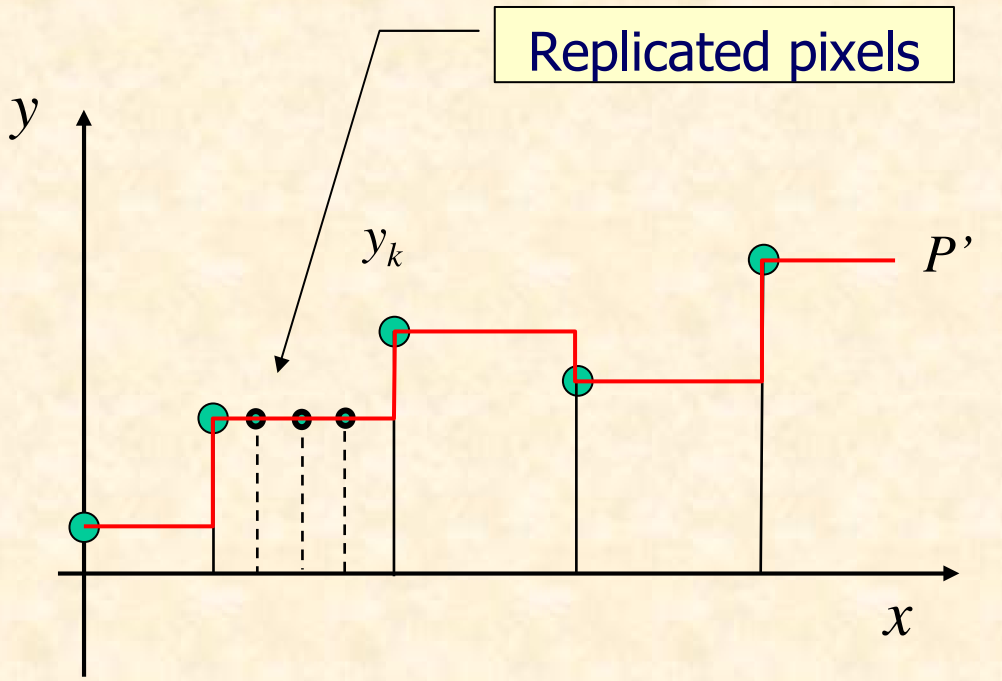
Bad idea

Image enlargement

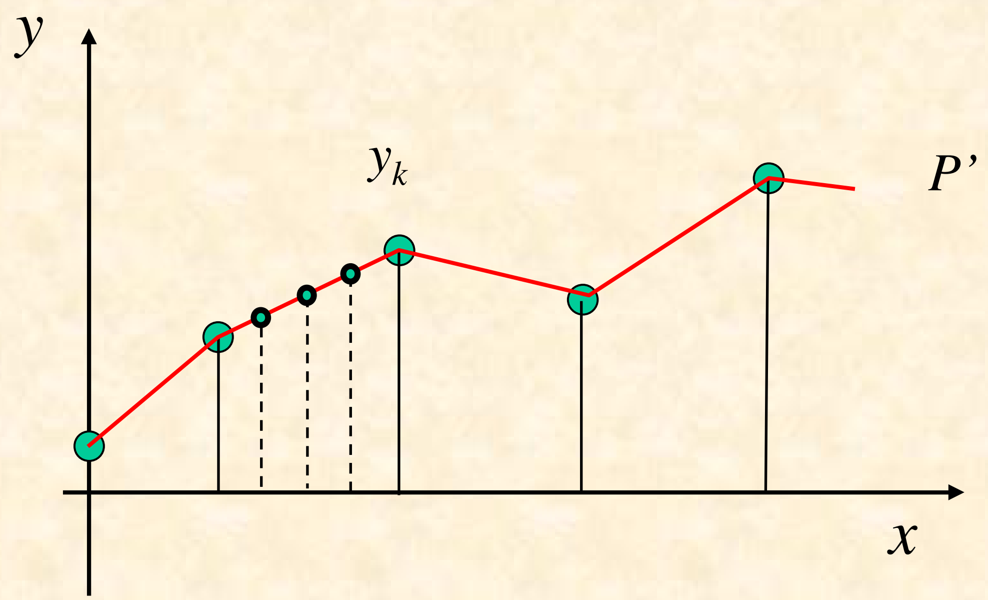
Interpolation – estimation of a function on the basis of its discrete samples (knots)
Image interpolation is also used in geometric transformations of images.



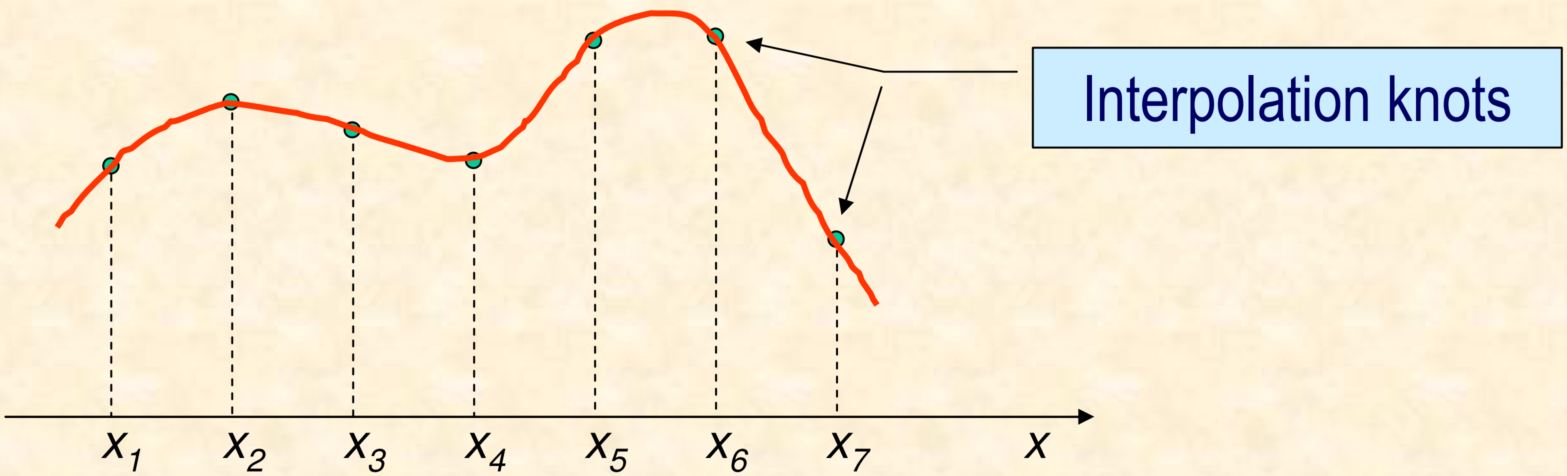
Zero-order interpolation



First-order interpolation



Polynomial interpolation

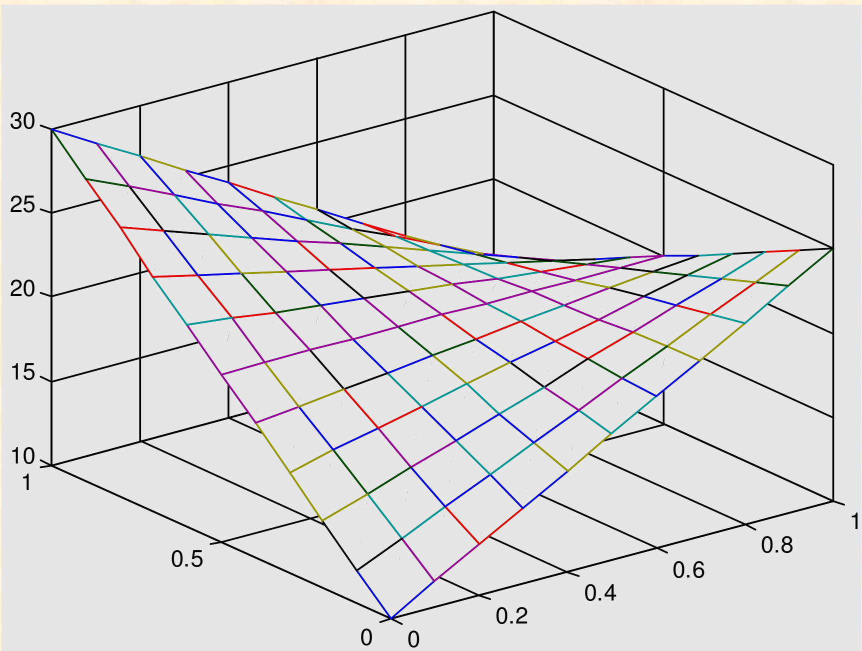


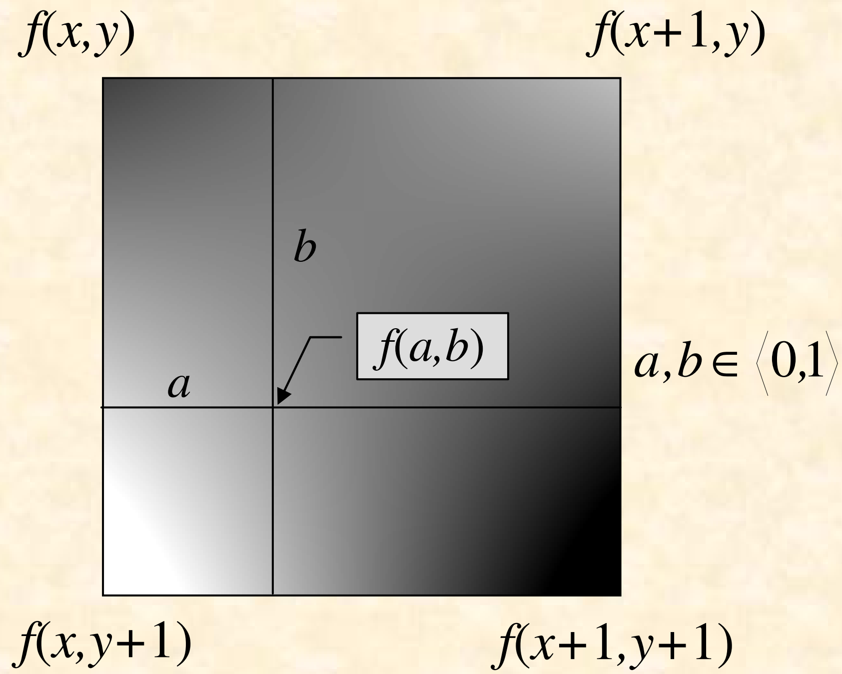
$$W(x)=a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

For given n knots and $f(x_i)$ for these knots, a polynomial $W(x)$ of order no larger than n is searched for so that:

$$W(x_i) = f(x_i).$$

Image enlargement by bilinear interpolation

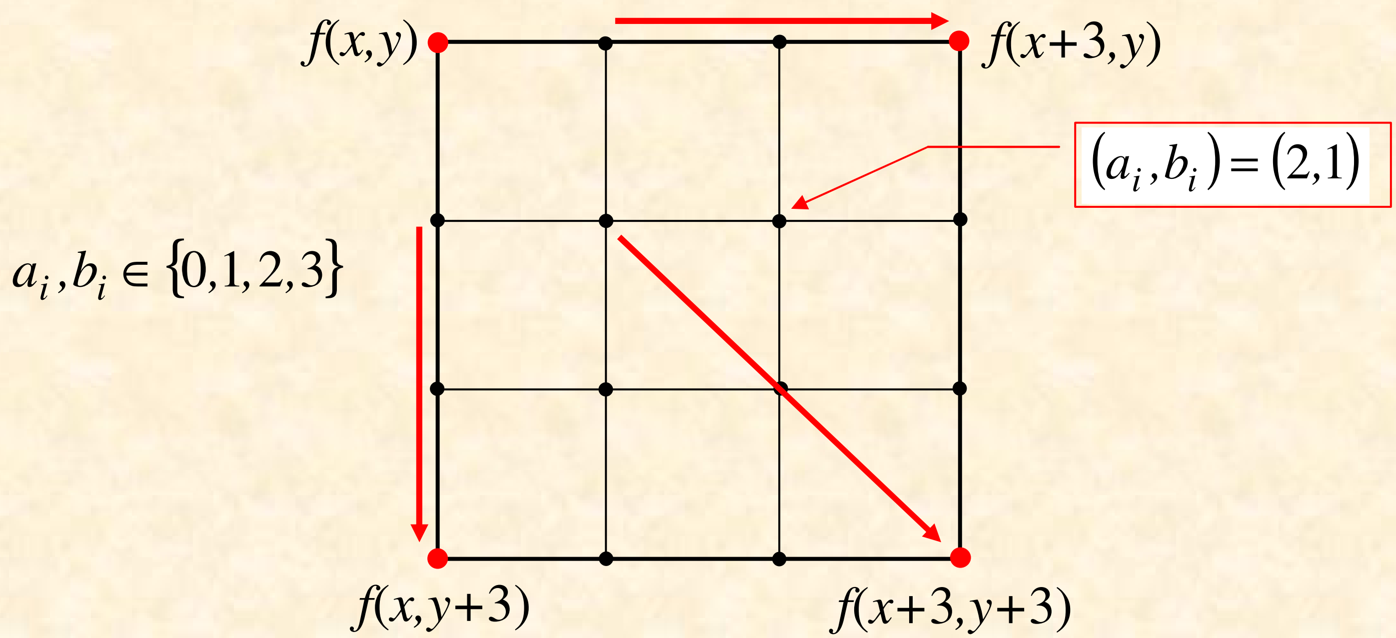




$$f(a,b)=(1-a)bf(x,y+1)+(1-a)(1-b)f(x,y)+a(1-b)f(x+1,y)+abf(x+1,y+1)$$

P. Strumiłło, M. Strzelecki

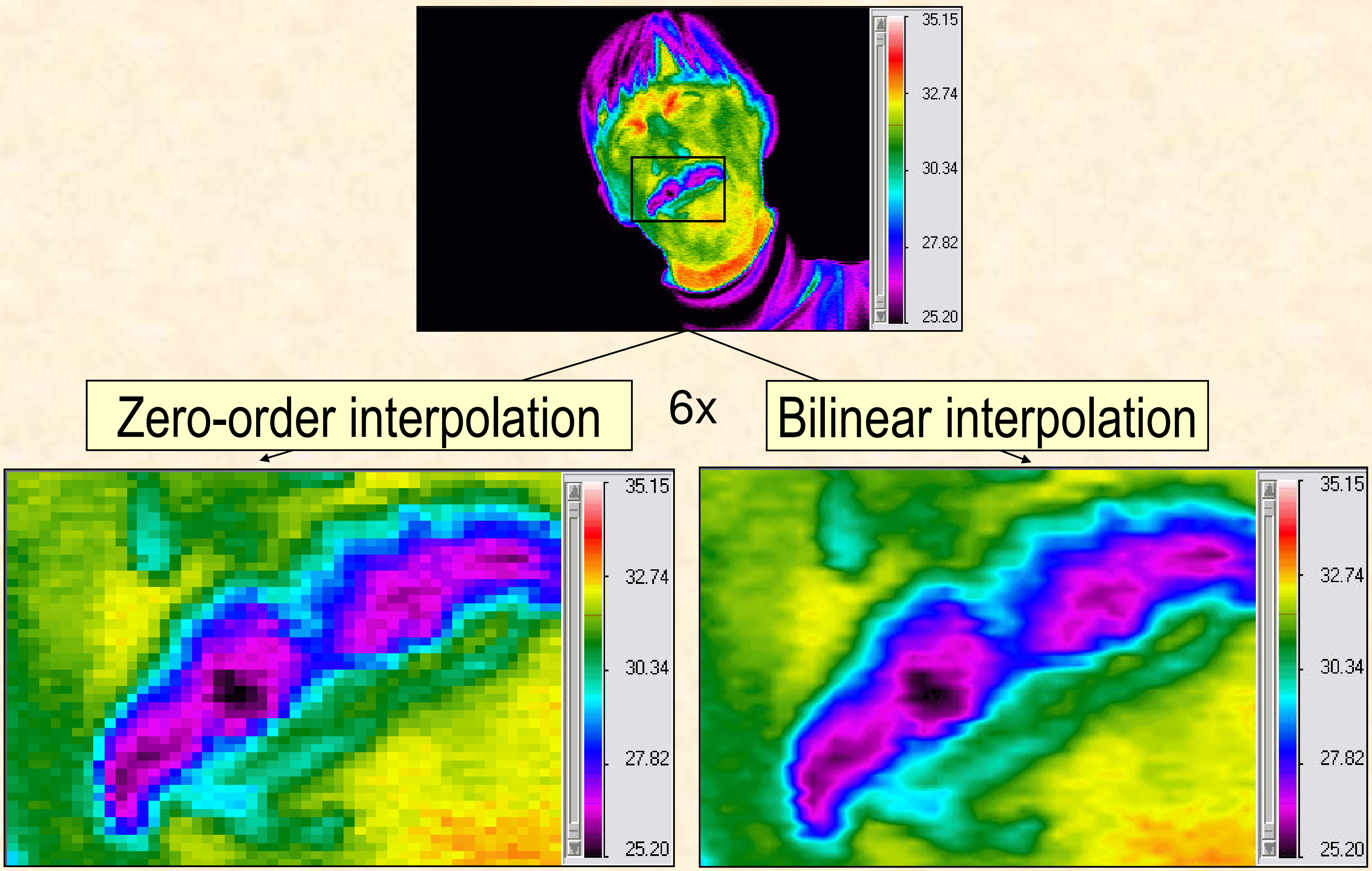
3x image enlargement



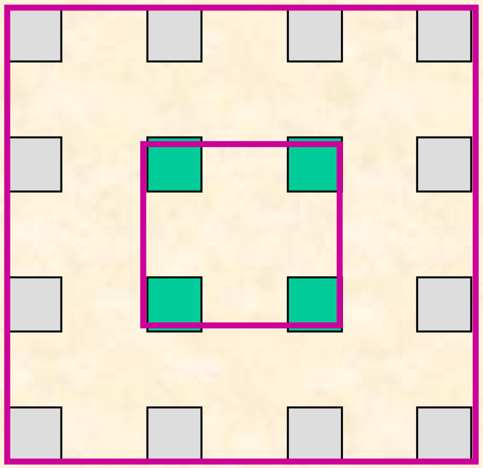
• - interpolated pixels

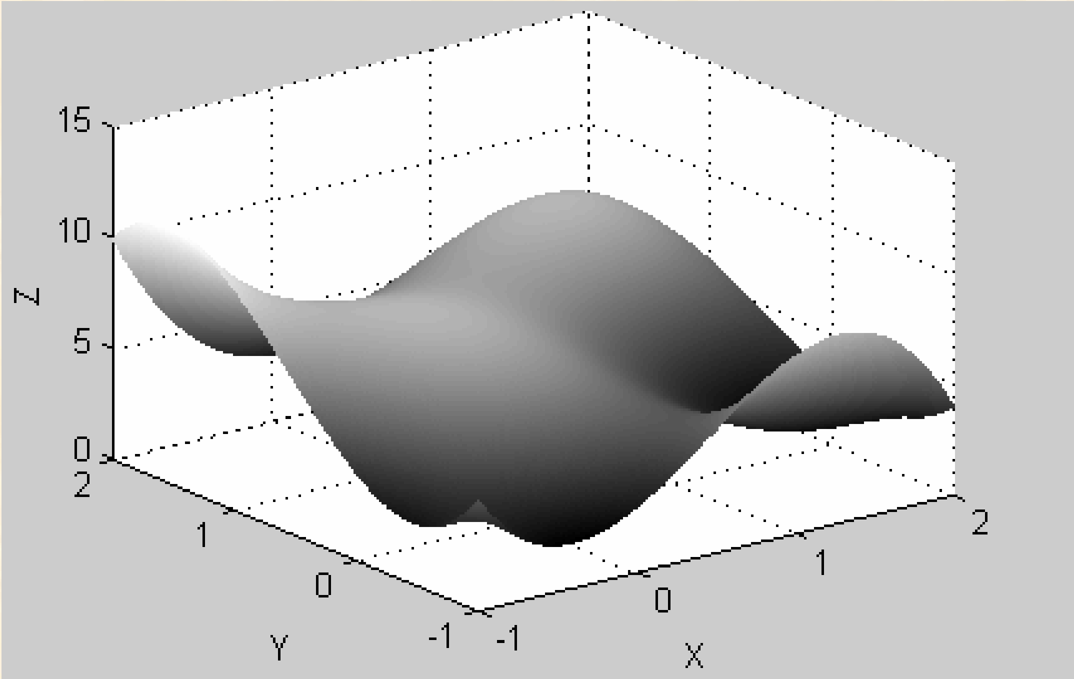
• - original image pixels

Image interpolation methods - comparison



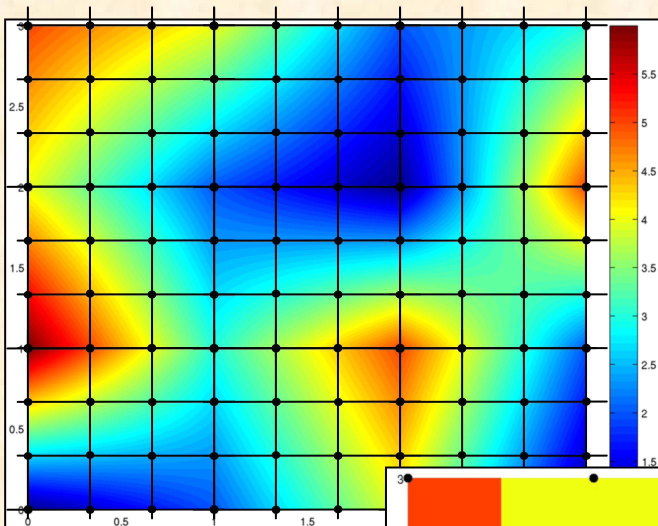
Bicubic polynomial interpolation



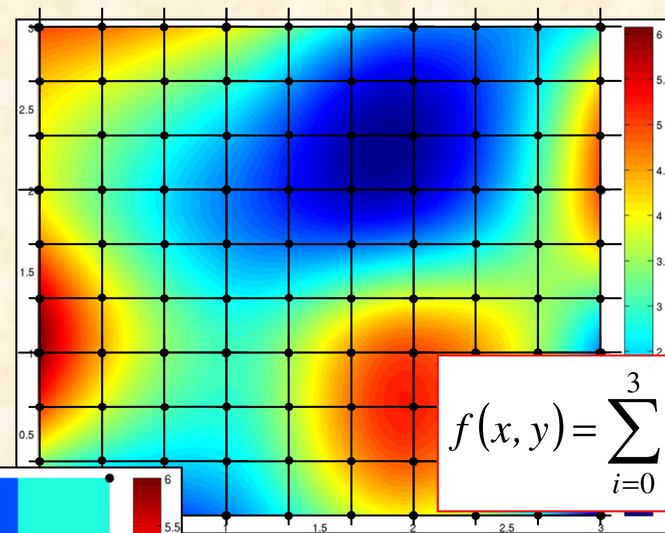


$$p(x,y)=\sum_{i=0}^{i=3}\sum_{j=0}^{j=3}a_{ij}x^i y^j$$

Image interpolation methods - comparison

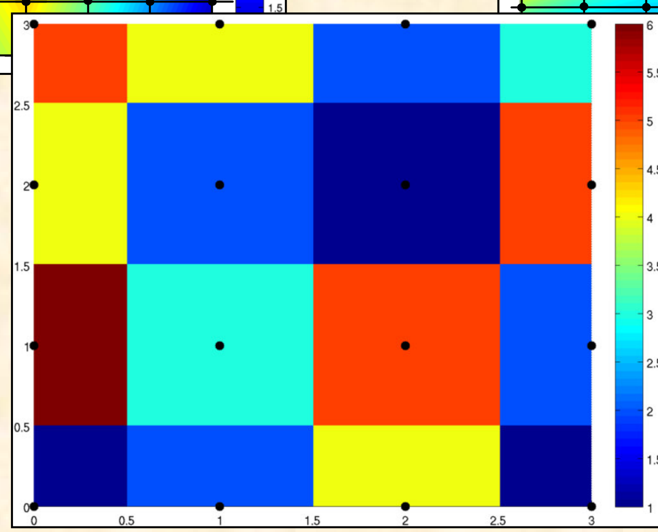


Bilinear



$$f(x,y)=\sum_{i=0}^{3}\sum_{j=0}^{3}a_{ij}x^{i}y^{i}$$

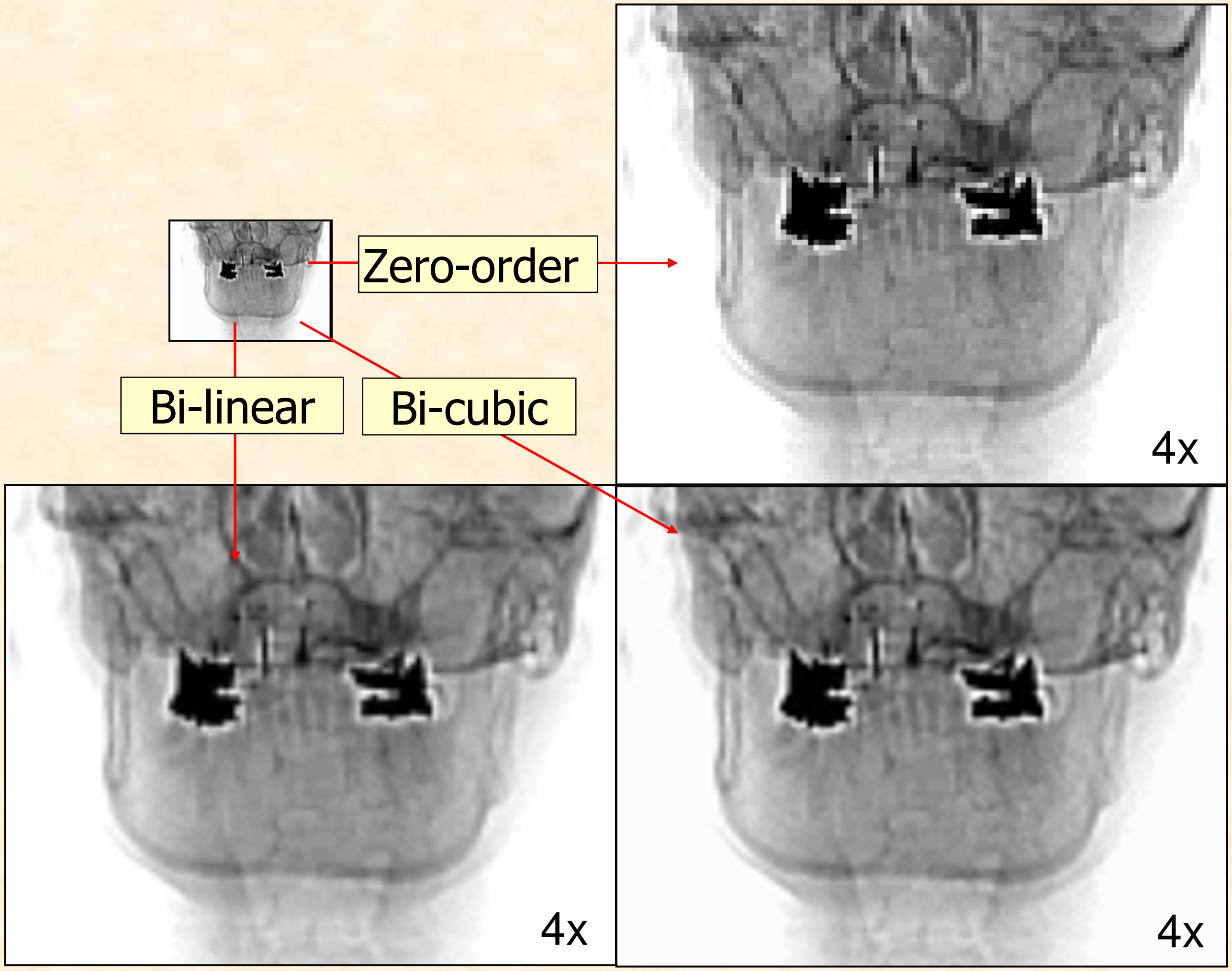
Bicubic – interpolation by 3-rd order polynomial



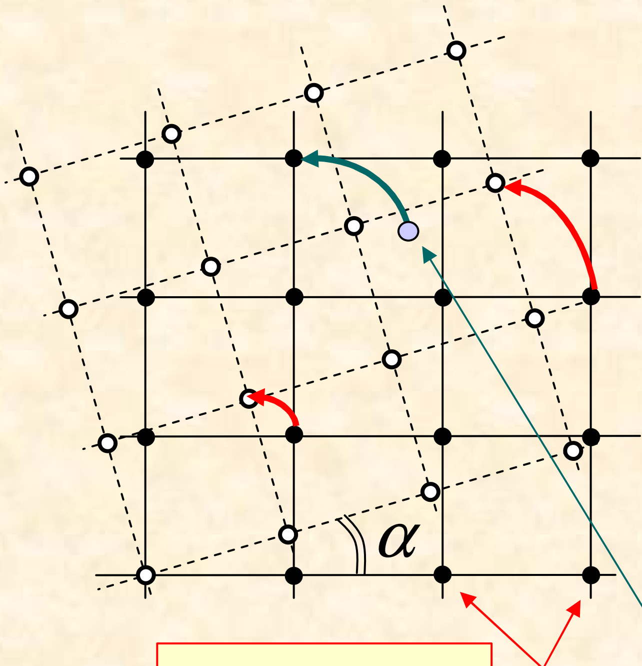
Pixel replication

Wikipedia

Image interpolation methods - comparison



Interpolation for image rotation



Note the distortions!

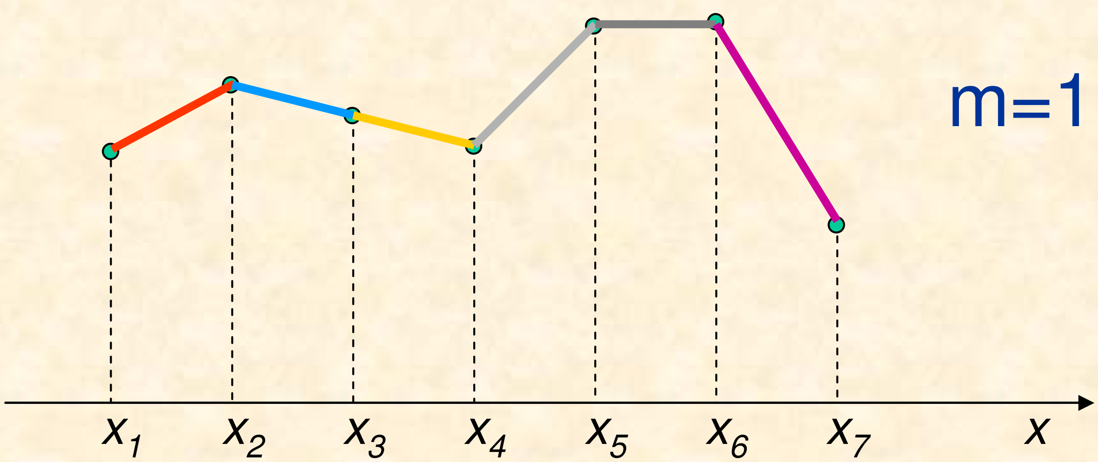
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{cases} x\cos\alpha - y\sin\alpha \\ x\sin\alpha + y\cos\alpha \end{cases}$$

x', y' is substituted from image raster and we find x, y, i.e. where the source image pixel should be taken from for display

Spline interpolation

Function S_m given on $x_0<x_1<x_2<\ldots<x_{n-1}<x_n$ is a spline of order m if:

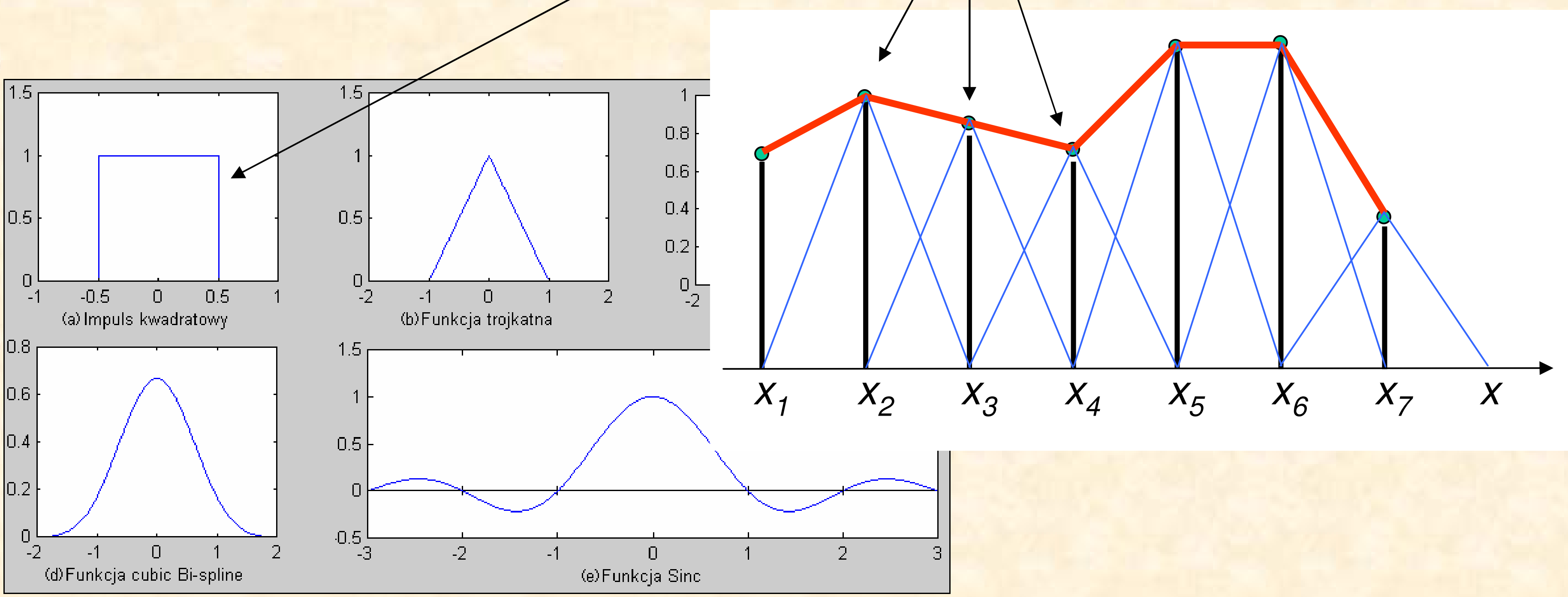
- it is a polynomial of order m or lower for each sub-range $<x_i, x_{i+1}>$
- S_m is of class C^{m-1} ,i.e. it is continuous with its derivatives in x_i (left and right hand-side limits are equal)



Spline interpolation

Interpolating function is obtained by convolution:

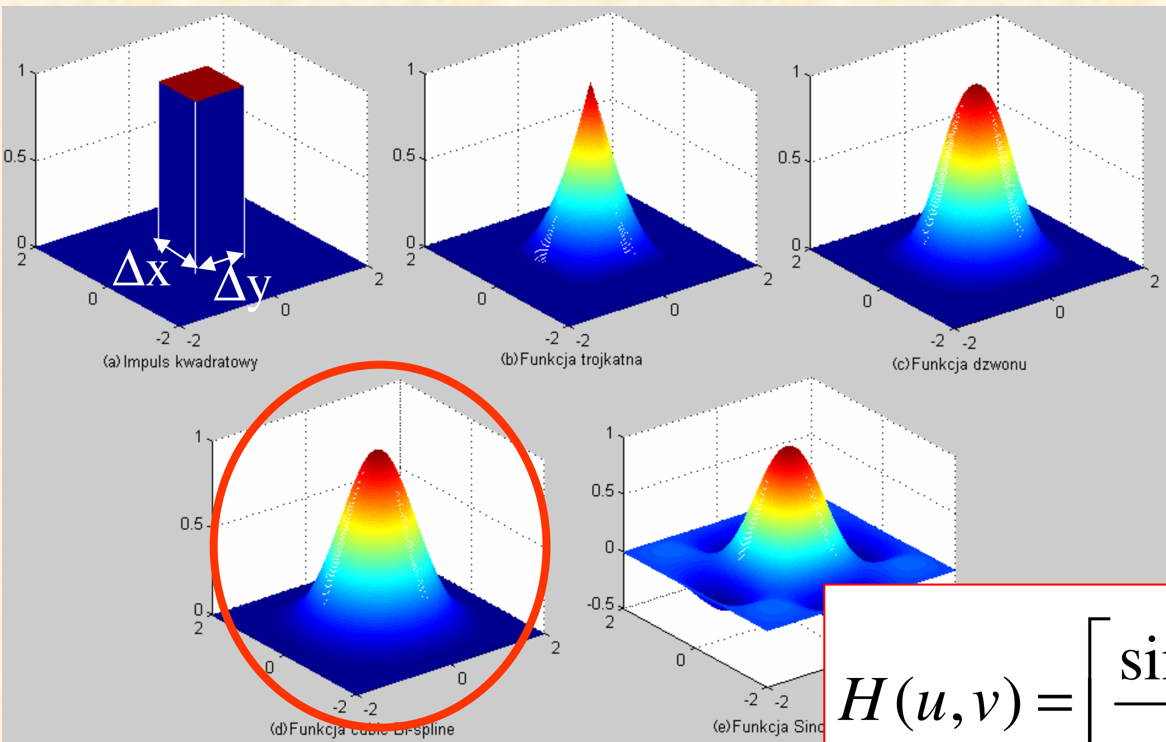
$$f_i(x) = S_m(x) * f(x_i)$$



2D spline interpolation by convolution

$$f_I(x,y) = h(x,y) * f(x,y)$$

$$F_I(u,v) = H(u,v)F(u,v)$$



cubic B-spline

$$H(u,v) = \left[\frac{\sin(\pi u \Delta x)\sin(\pi v \Delta y)}{(\pi u \Delta x)(\pi v \Delta y)} \right]^4$$

Image enlargement

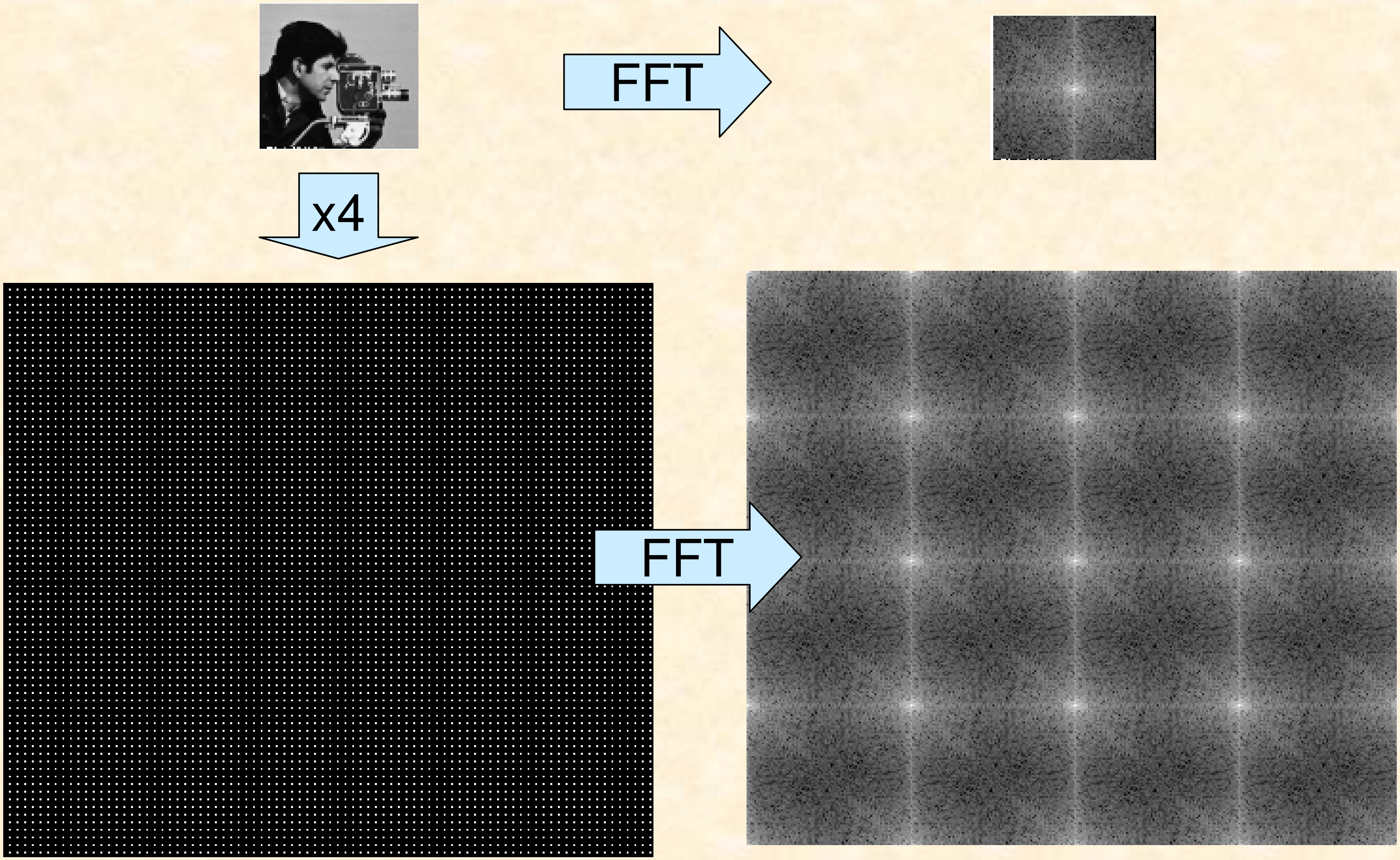
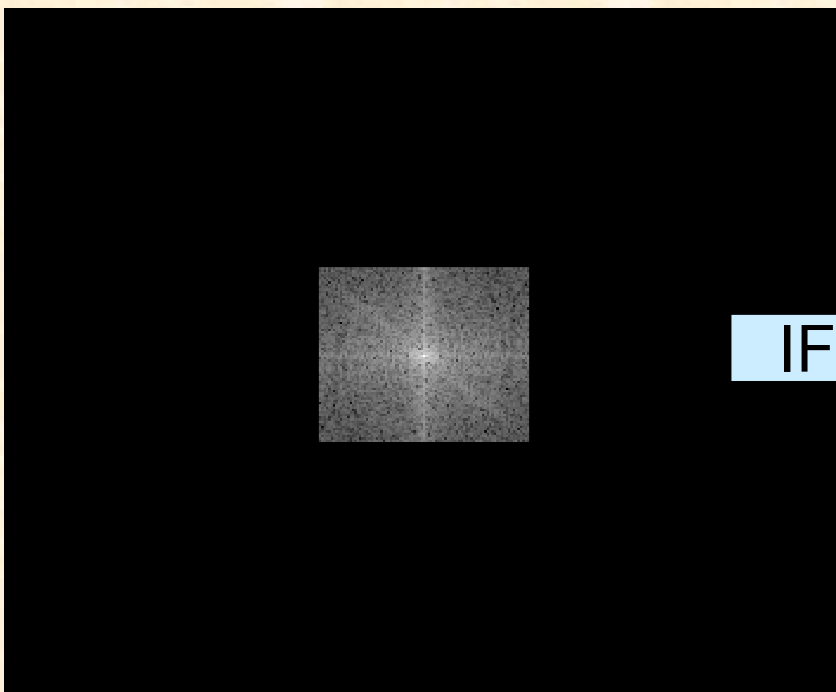


Image enlargement

x4



IFFT

Image interpolation - comparison

***P**eak*
***S**ignal to*
***N**oise*
***R**atio*

$$PSNR = 10\log_{10}\left[\frac{255^2 MN}{\sum_{i=1}^{M}\sum_{j=1}^{N}[f(i,j) - f_{INT}(i,j)]^2}\right]$$

Interpolation method	*PSNR* [dB]
pixel replication	31.6371
bilinear	35.0411
bicubic	35.5752
cubic B-spline	35.7082

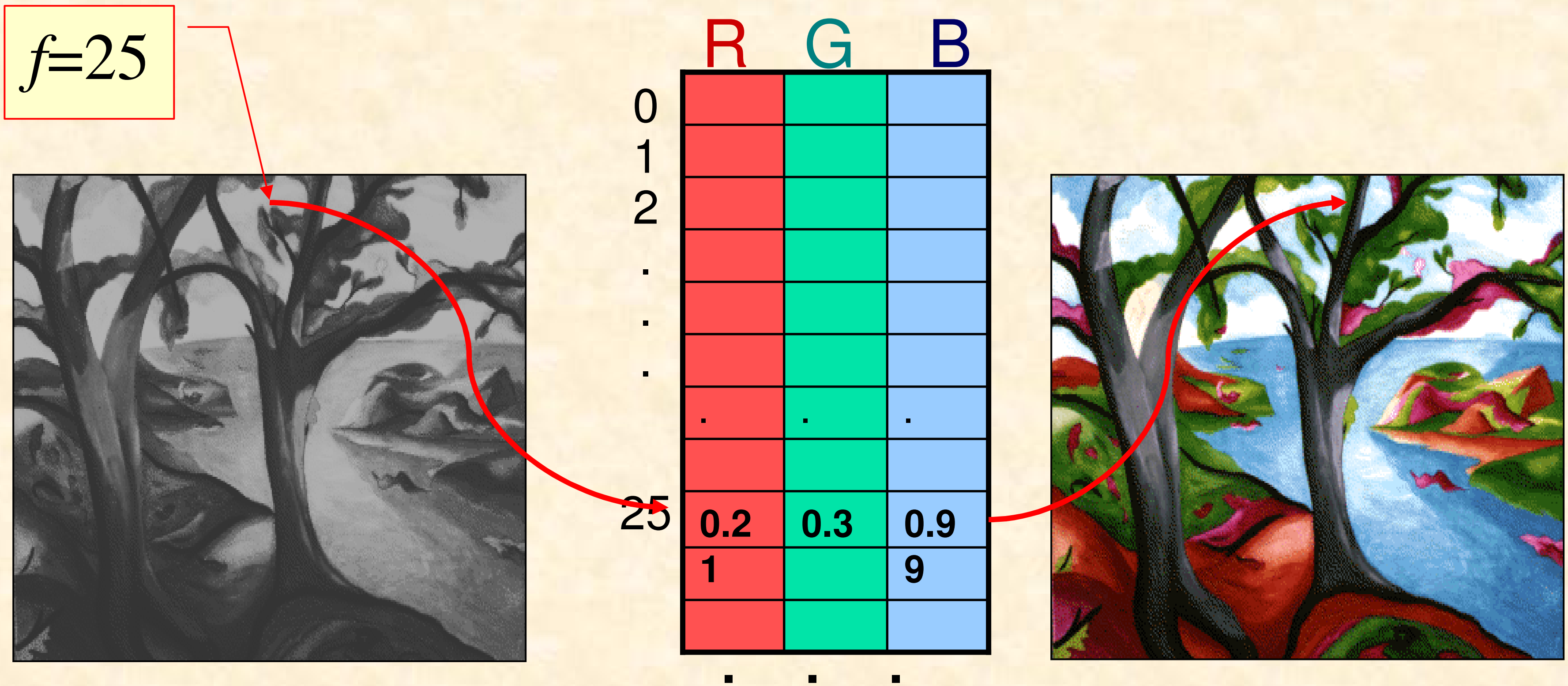
Image enhancement by pseudocoloring

Selected gray level range is replaced by a colour predefined in the look-up-table

```
lut : array[0..L-1] of longint;
f :  array[0..N-1 ,0..N-1] of longint;

for i:=0 to N-1 do for j:=0 to N-1 do
        f[i,j]:=lut[f[i,j]];
```

Colour indexed image



Monochrome image

Colour palette (look-up table)

Colour image

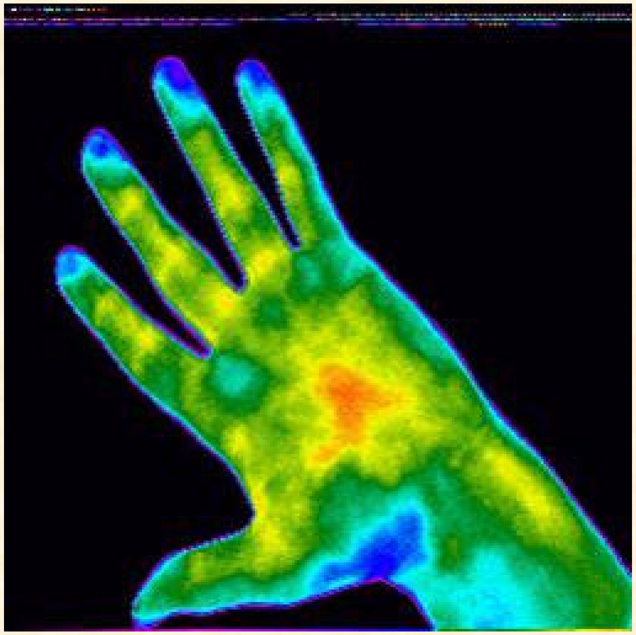
P. Strumiłło, M. Strzelecki

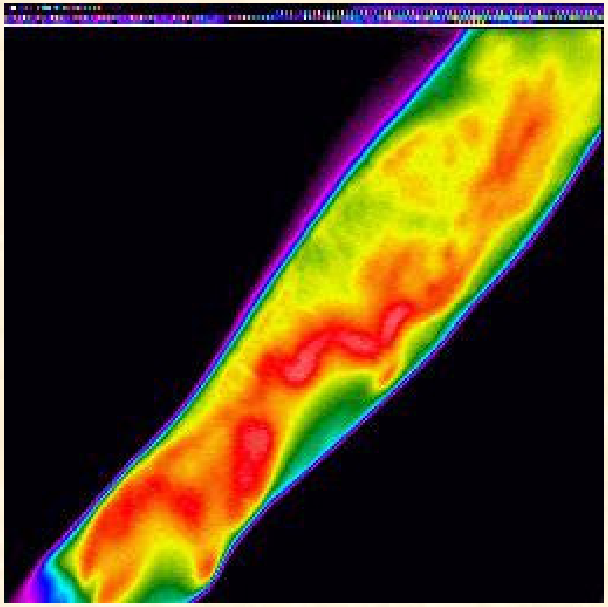
Image pseudocoloring –application examples

Image pseudocoloring –application examples

[A. Weeks, Fundamentals of Electronic Image Processing, IEEE Press, 1996]

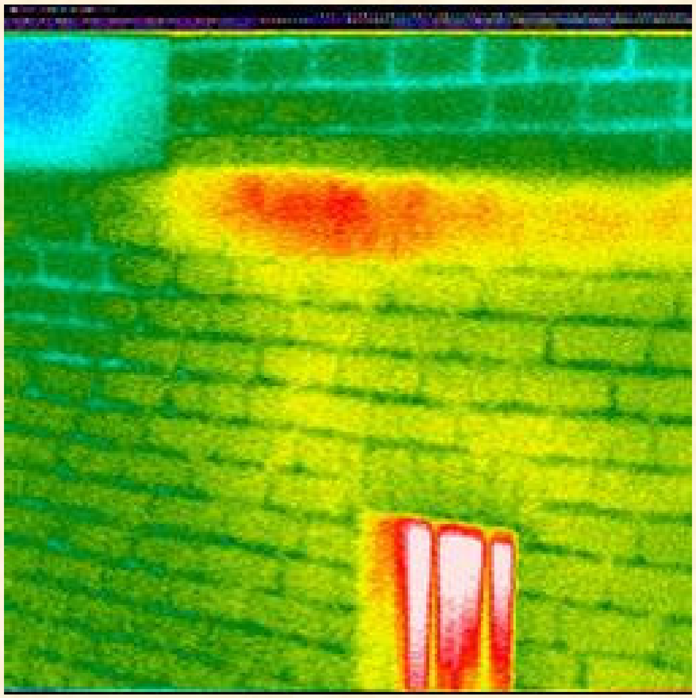
Image pseudocoloring –application examples

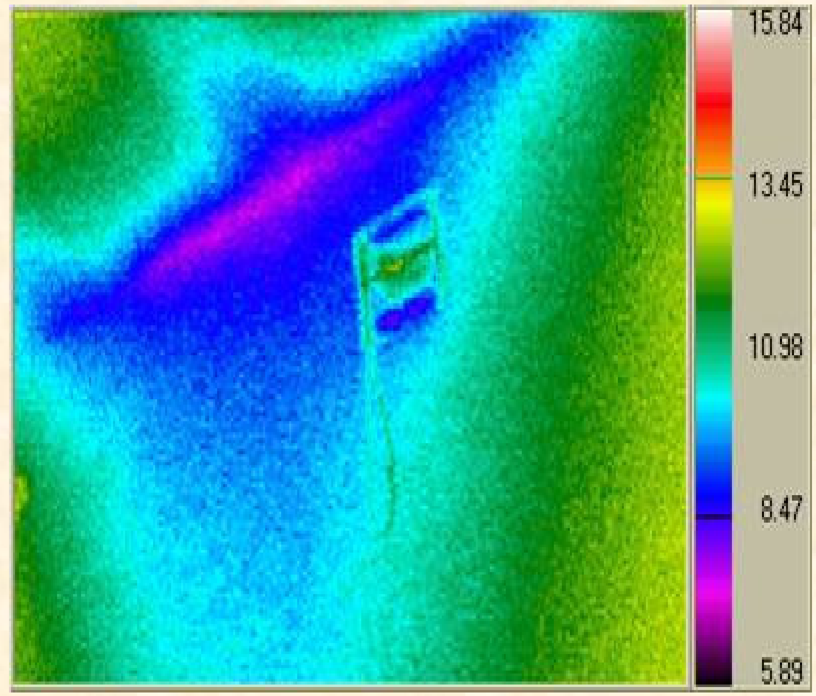




Allergy
[Pracownia Termografii, IE]

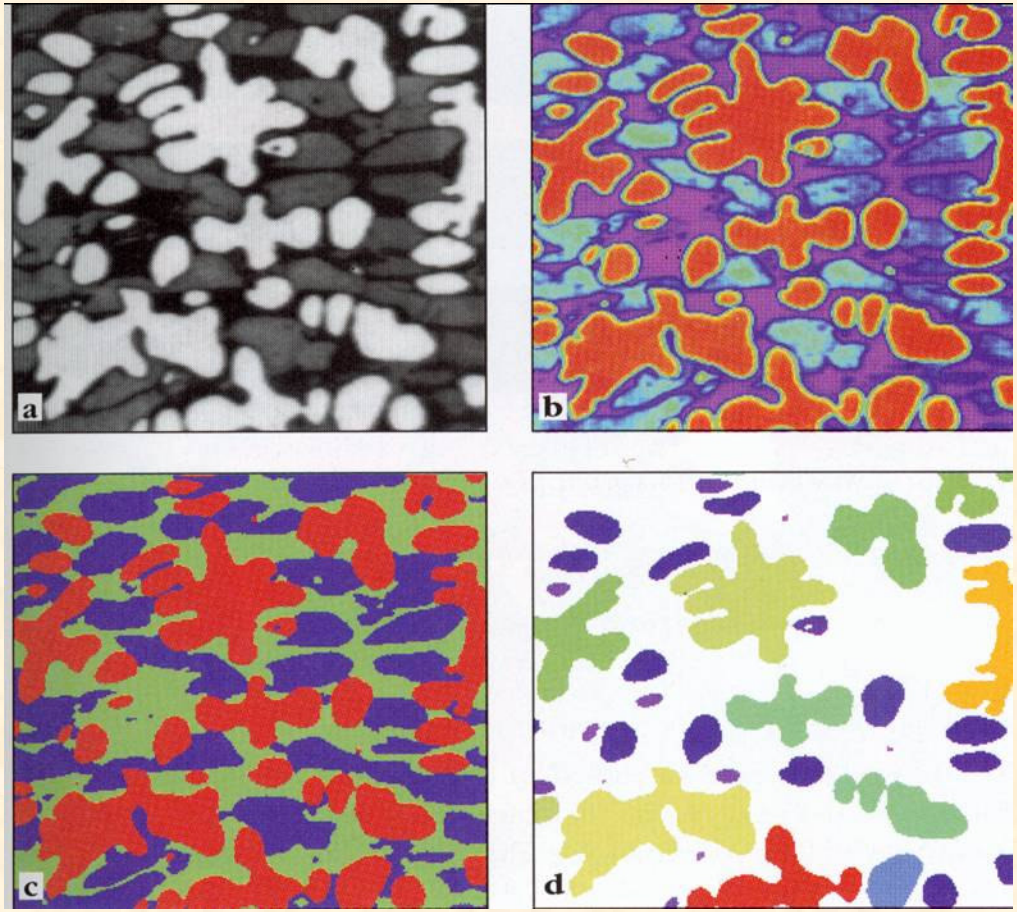
Image pseudocoloring –application examples





Infrared images of buildings (testing of thermal insulation quality)
[Thermography Lab, Institute of Electronics]

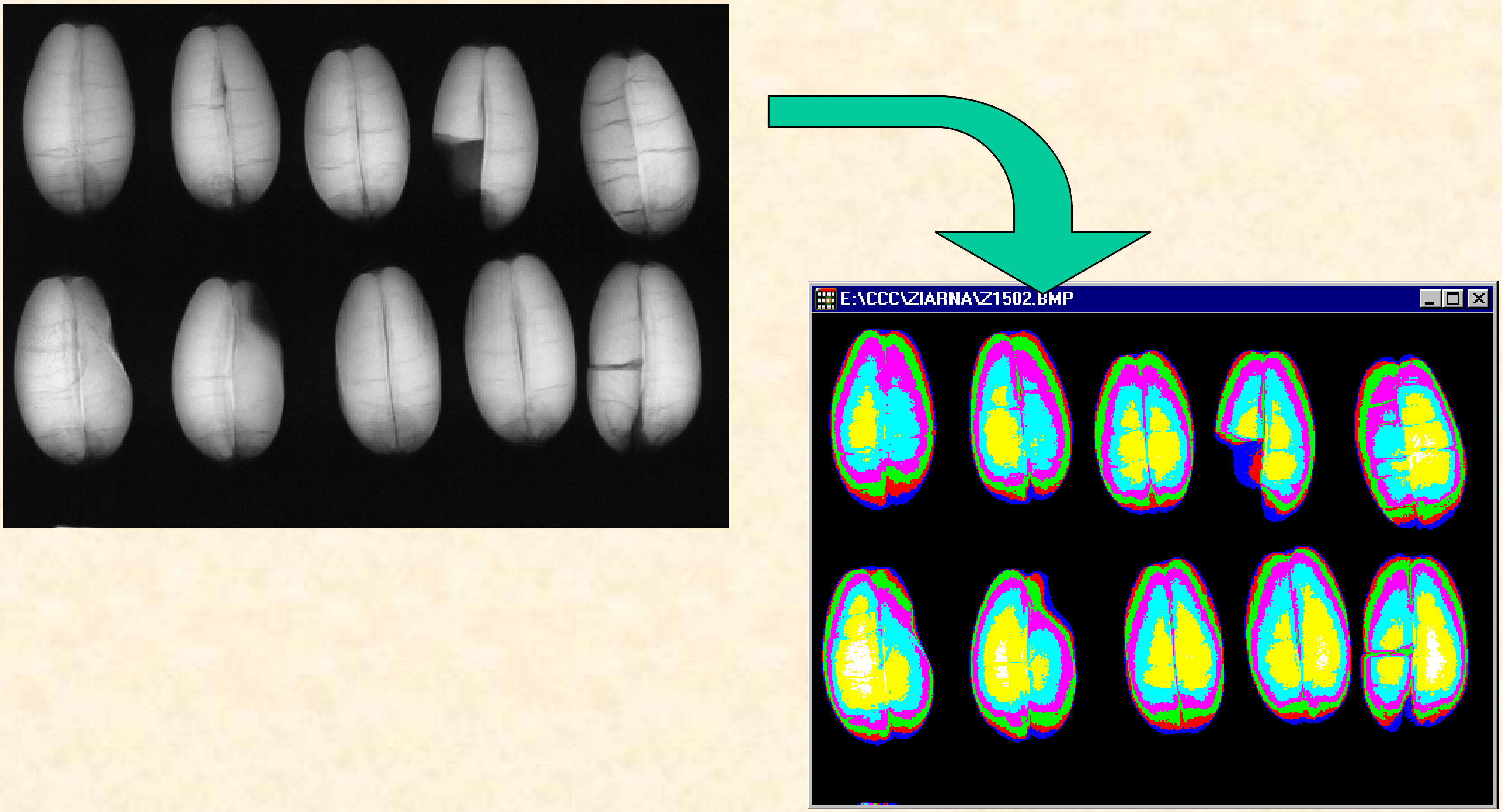
Image pseudocoloring –application examples



Metallographic images

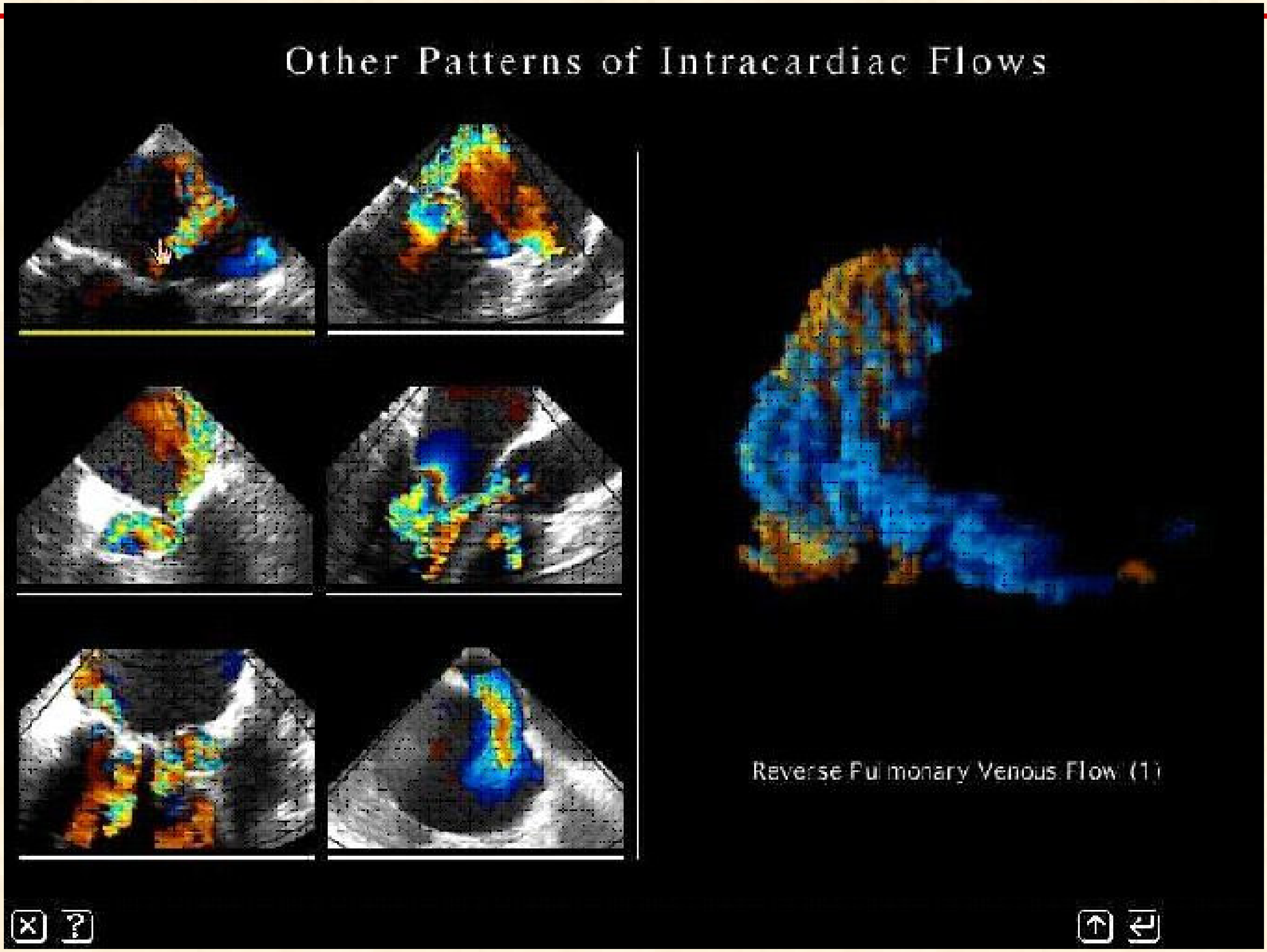
[J. Russ, The Image Processing of Handbook, CRC Press 1995]

Image pseudocoloring –application examples



X ray images of wheat grains [Instytut Agrotechniki PAN, Lublin]

Doppler ultrasonography



Cardiac images [Medical and Biological Informatics, GCRC, Heidelberg, Germany]