Image filtering

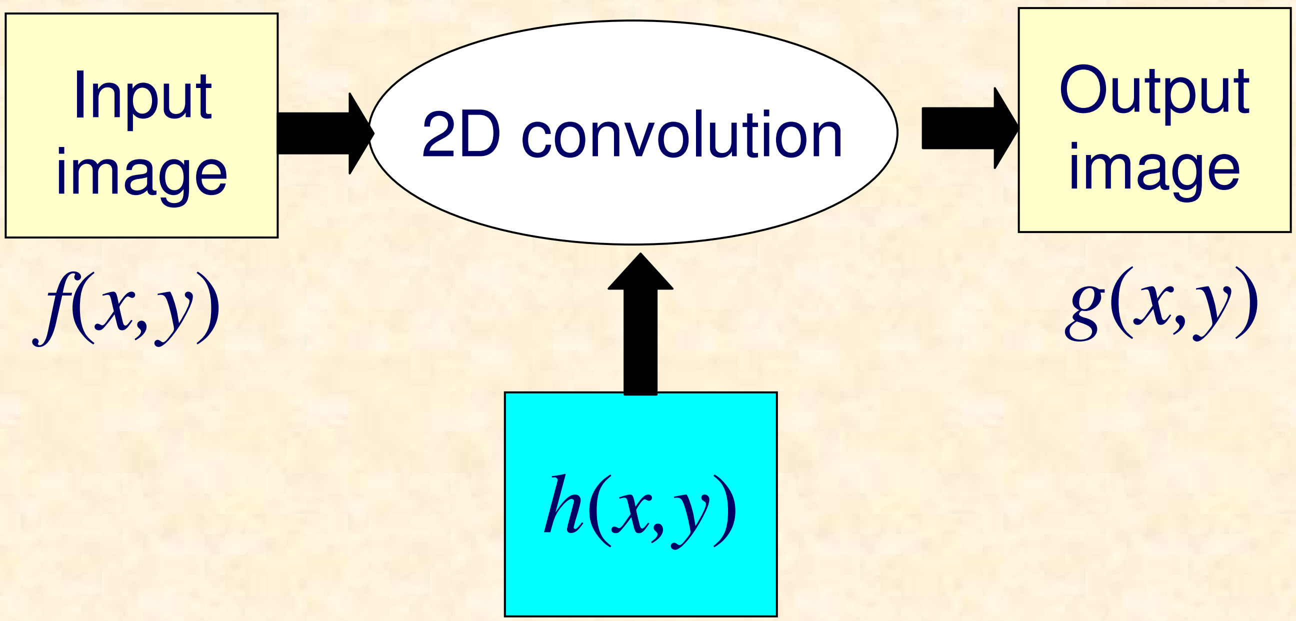
In Fourier domain

In spatial domain

Linear filters

Non-linear filters

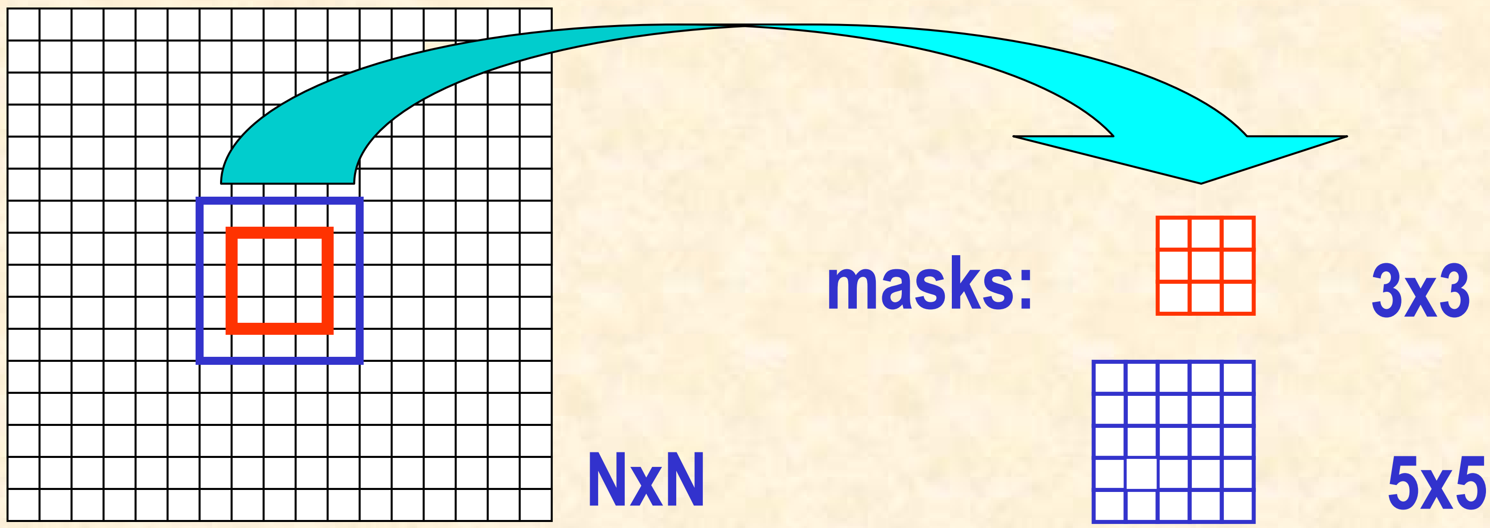
Image filtering in spatial domain



$$g(x,y) = IF\{\ H(u,v)\ F\{f(x,y)\}\ \} =$$

$$IF\ \{H(u,v)\} ** IF\ \{F\ \{f(x,y)\}\ \} =$$
$$h(x,y) ** f(x,y)$$

Filter definition in spatial domain

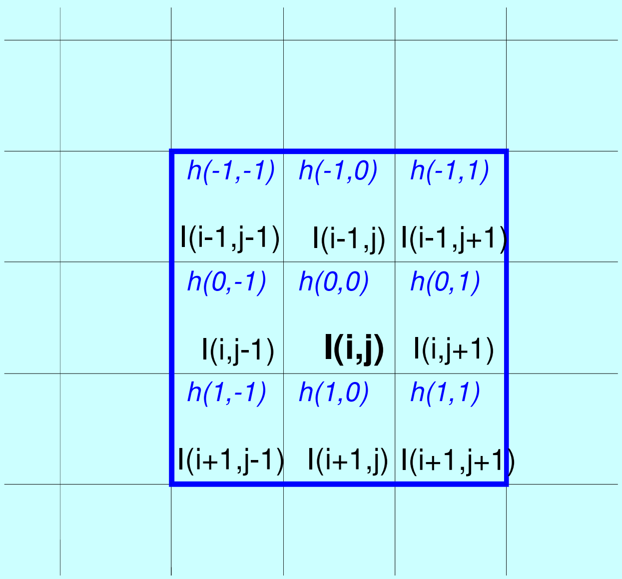


$\hat{h}(x,y)$

$$IF\{H(u,v)\}=h(x,y)$$

$\hat{h}$ is selected so that $F(\hat{h}(x,y)) = \hat{H}(x,y) \approx H(x,y)$

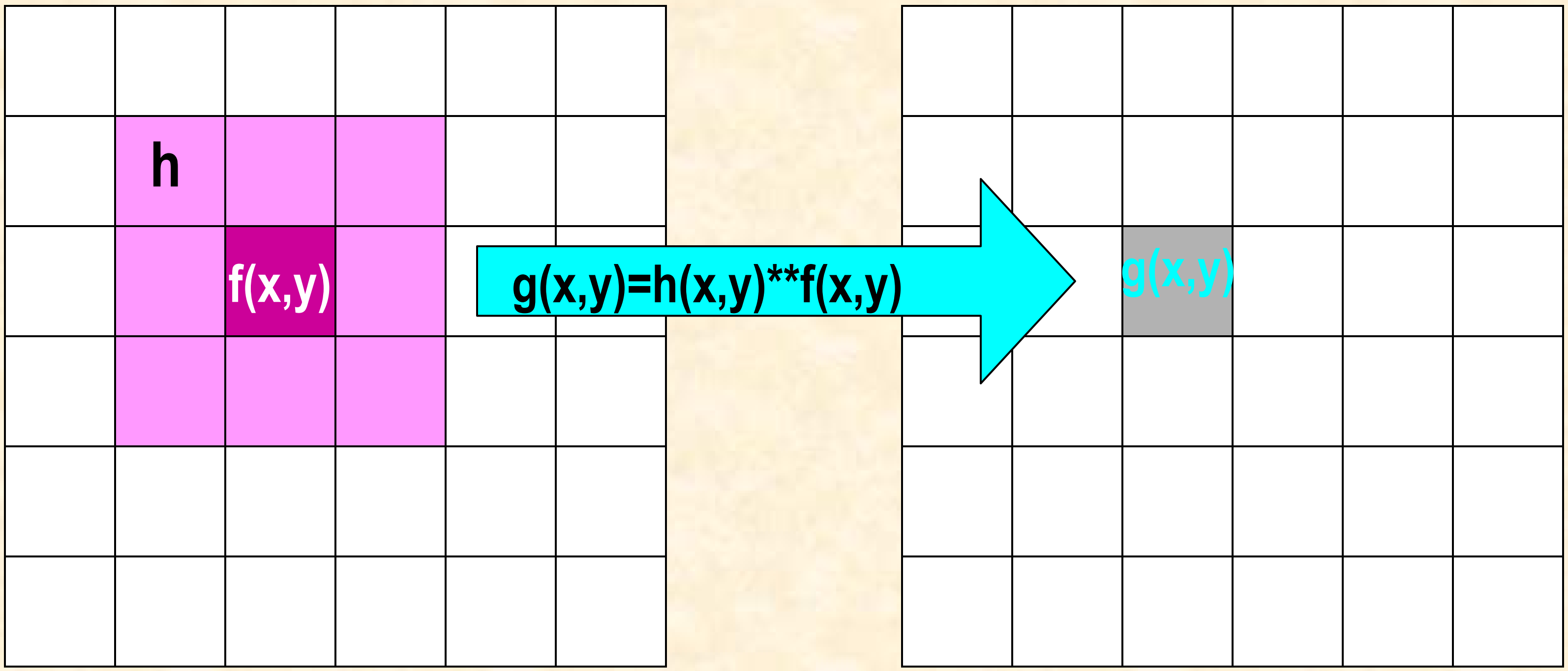
Image and the filer mask convolution



$$
\begin{aligned}
G(i,j) = {} & \\
& I(i-1,j-1)h(-1,-1) + I(i-1,j)h(-1,0) + I(i-1,j+1)h(-1,1) + \\
& I(i,j-1)h(0,-1) + \mathbf{I(i,j)}h(0,0) + I(i,j+1)h(0,1) + \\
& I(i+1,j-1)h(1,-1) + I(i+1,j)h(1,0) + I(i+1,j+1)h(1,1)
\end{aligned}
$$

This is true for symmetric masks only !

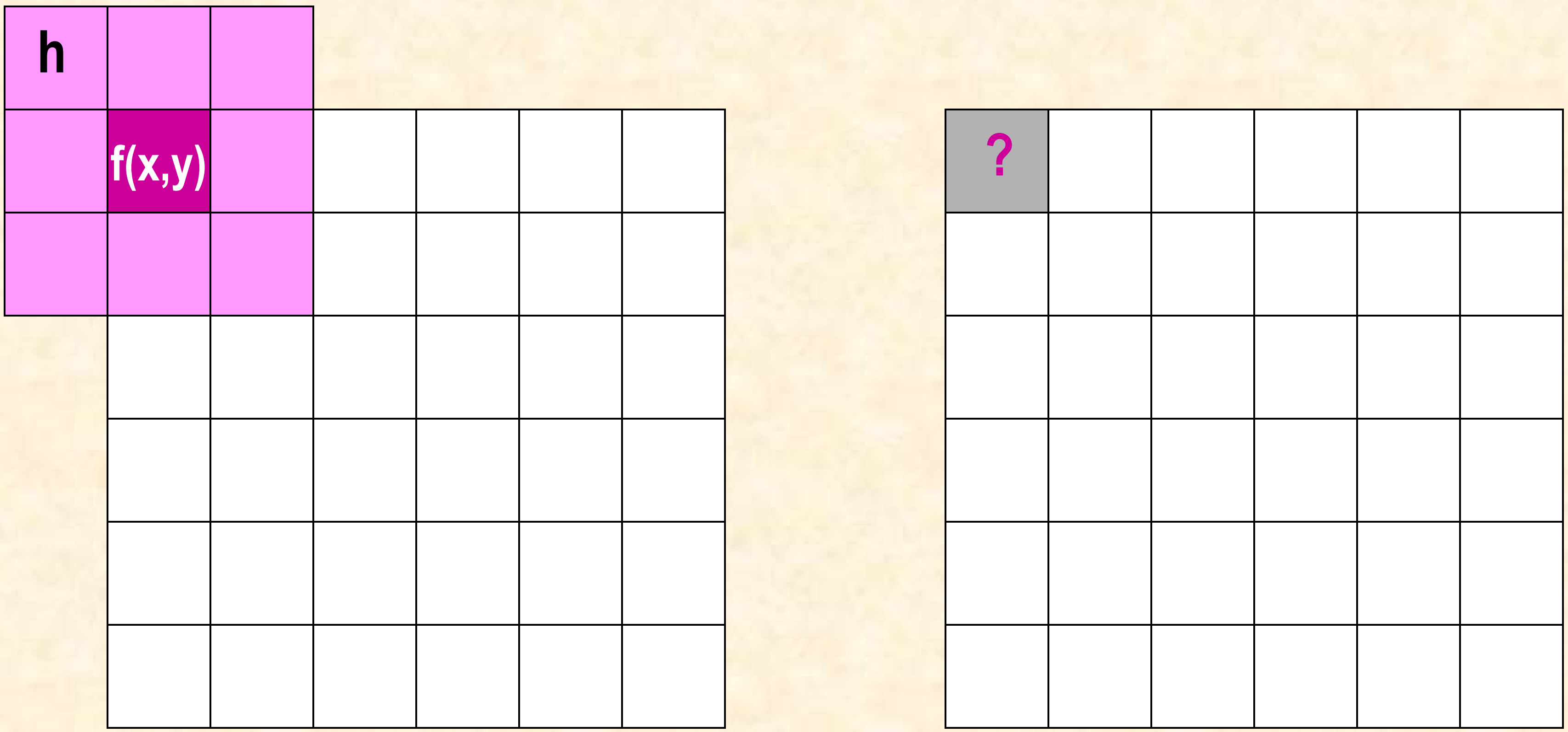
Computing the filtered image



source image *f*

output image *g*

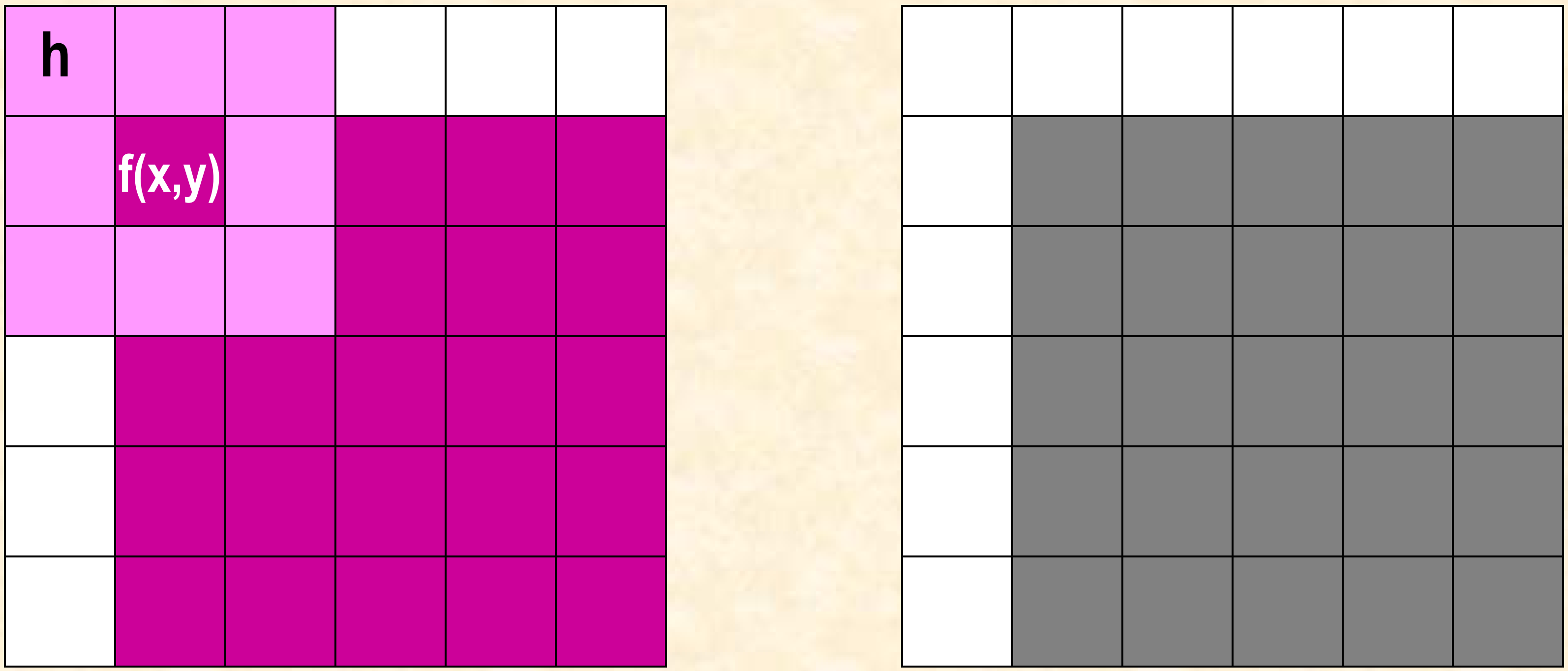
Boundary effects



source image *f*

output image *g*

Boundary effects – 3x3 mask



Boundary columns and rows of (NxN) image are neglected and the filtered image is of size (N-2)x(N-2)

Image filtering – the algorithm

```
f, g : array[0..N-1, 0..N-1] of byte;
{ size2 – half size of the mask}
h : array[-size2..size2,-size2..size2] of integer;
...
for i:=1 to N-2 do  for j:=1 to N-2 do
        begin
           g[i,j]:=0;
           for k:=-size2 to size2 do  for l:=-size2 to size2 do
                 g[i,j]:=g[i,j] + f[i+k,j+l] * h[i+k,j+l];
        end;
...
```

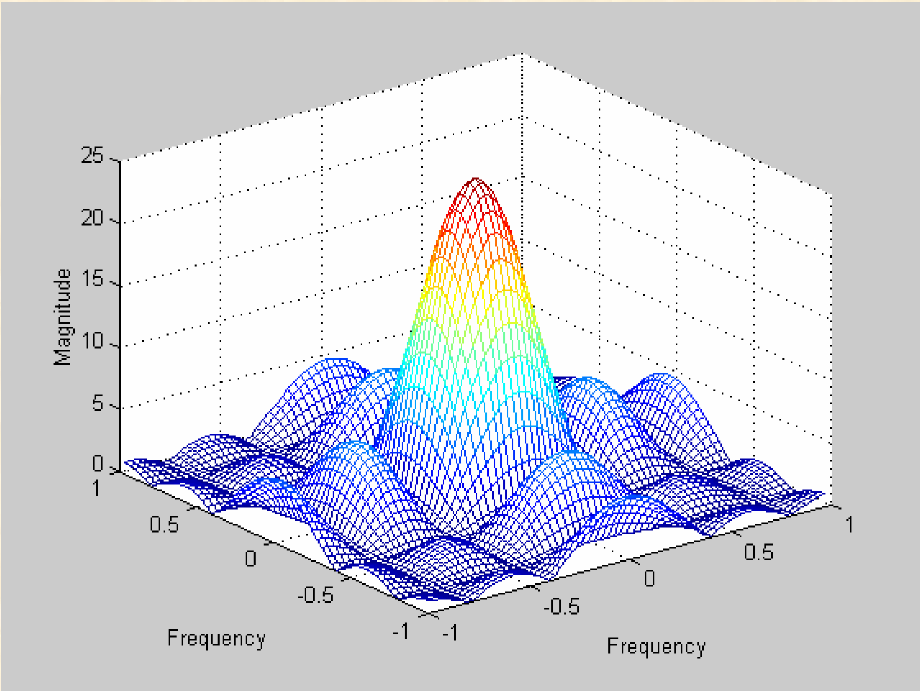
Range check g[i,j] !!!

Low pass filter

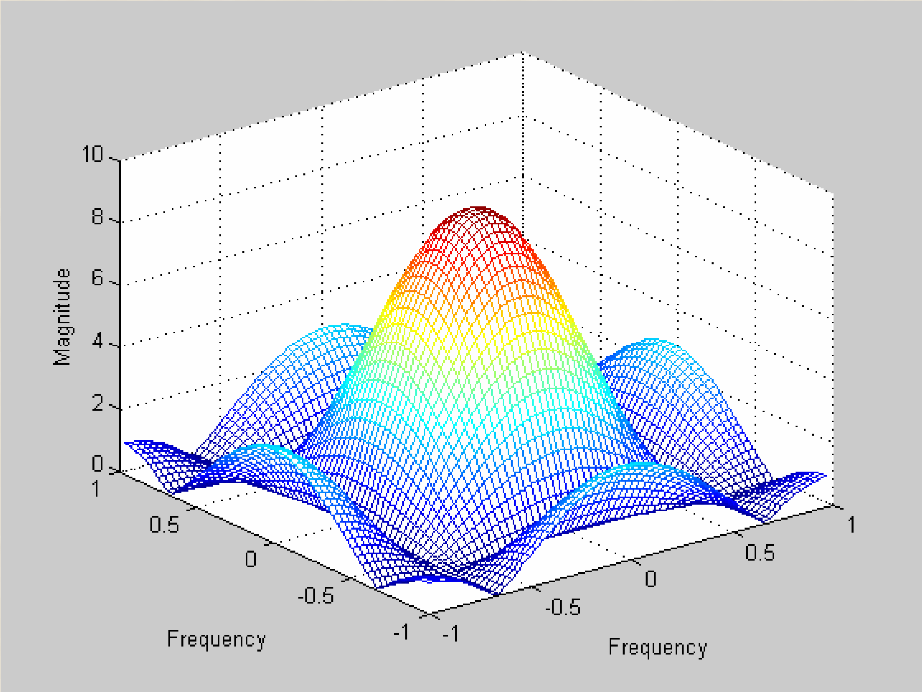
$$h_1 = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad h_2 = \frac{1}{25}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Can one use mask of even size ?

Frequency characteristics of low pass filters



for 5x5 mask



for 3x3 mask

H=freqz(h,m,n)

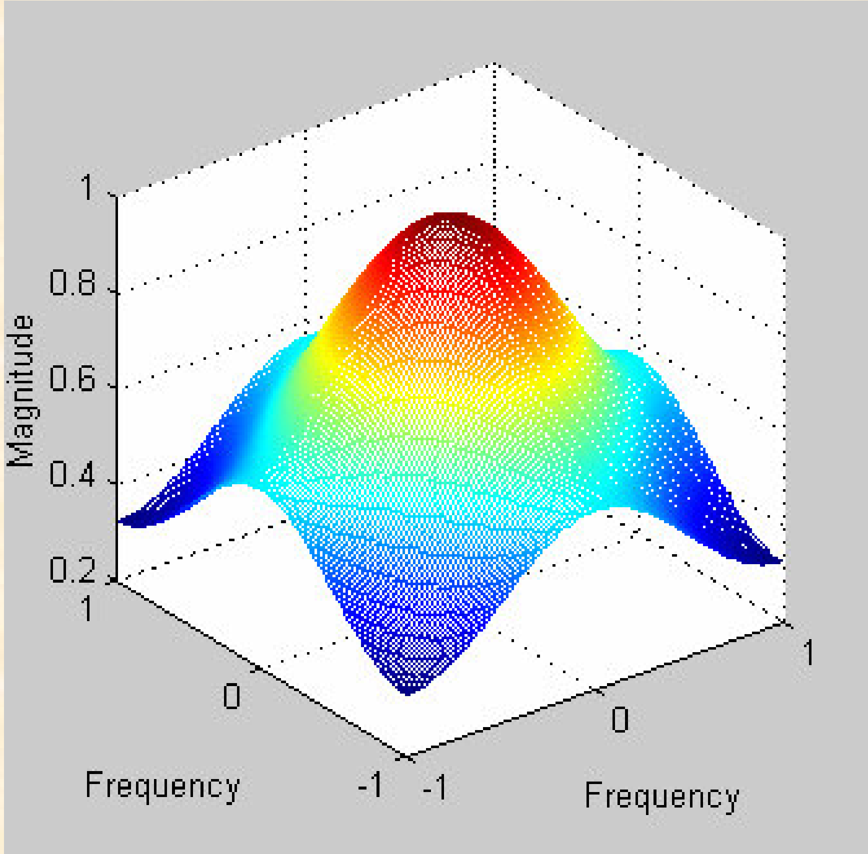
Low-pass filtering the image

Source image

3x3 mask

5x5 mask

Gaussian filter



$$h = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$h(x, y) = e^{\frac{-\pi(x^2+y^2)}{{d_0}^2}}$$

$$H(u, v) = e^{\frac{-\pi {d_0}^2 (u^2+v^2)}{N}}$$

Image filtering using the Gaussian filter

source image

filtered image

Image low-pass filters - examples

Image distorted by the Gaussin noise N(0, 0.01)

for grayscale <0,1>

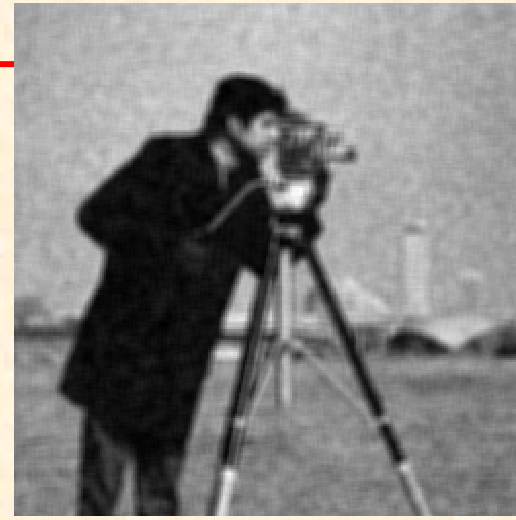
Low pass filter 3x3

Gaussian filter 3x3

Butterworth filter D_0=50

Image low-pass filters - examples

Image distorted by the Gaussian noise N(0, 0.01)



low-pass filter 5x5

Gaussian filter 5x5

Butterworth filter D_0=30

Image low-pass filters - examples

Image distorted by the Gaussian noise N(0, 0.002)

Low pass filter 3x3

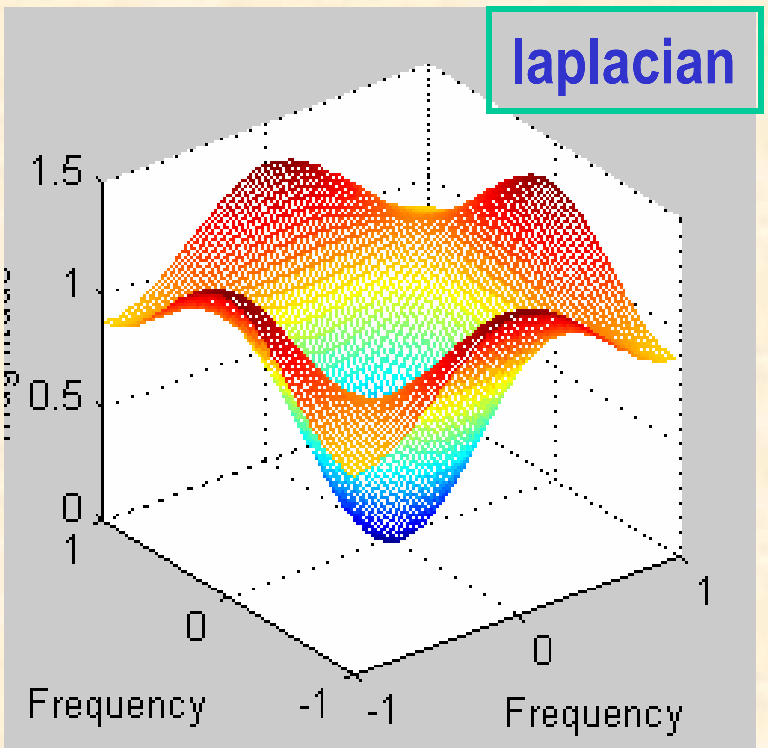
Gaussian filter 3x3

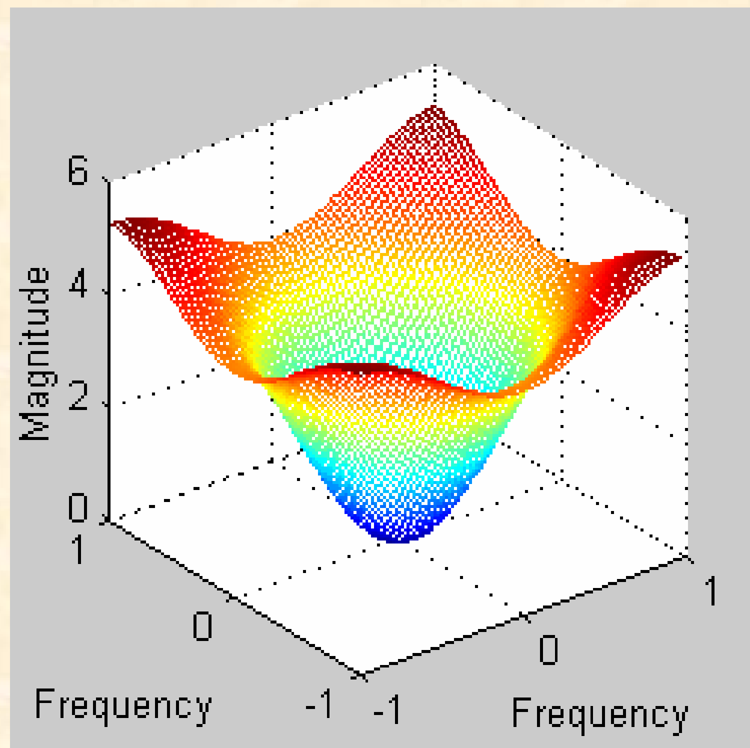
Butterworth filter D_0=50

High-pass filters (derivative filters)

$$h_1 = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

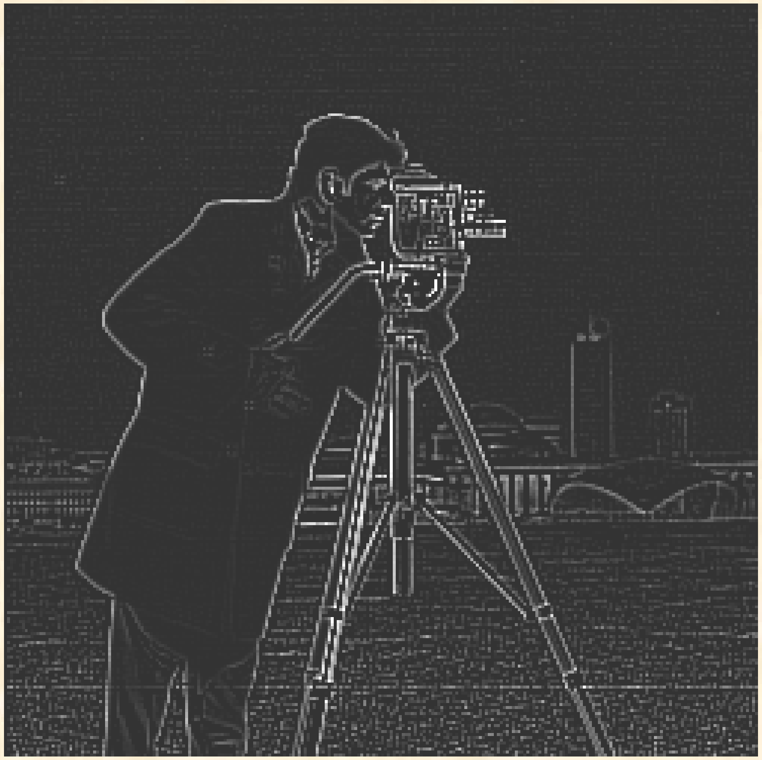
$$h_2 = \begin{bmatrix} 0.17 & 0.67 & 0.17 \\ 0.67 & -3.33 & 0.67 \\ 0.17 & 0.67 & 0.17 \end{bmatrix}$$





High-pass filtering the image

mask h_1



mask h_2

The „high boost” filter

$$f(x,y) = f_L(x,y) + f_H(x,y)$$

$$f_{HB}(x,y) = Af(x,y) - f_L(x,y) =$$
$$= (A-1)f(x,y) + f(x,y) - f_L(x,y) =$$
$$= (A-1)f(x,y) + f_H(x,y), \qquad A \geq 1$$

A=?

$$h_{HB} = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9A-1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

High boost filter - example

Laplace filter

A=1.1

A=1.5

A modified Laplace filter

$$h_2 = \begin{bmatrix} 0.17 & 0.67 & 0.17 \\ 0.67 & -3.33 & 0.67 \\ 0.17 & 0.67 & 0.17 \end{bmatrix}$$

$$h'_2 = \begin{bmatrix} 0.17 & 0.67 & 0.17 \\ 0.67 & -2.33 & 0.67 \\ 0.17 & 0.67 & 0.17 \end{bmatrix}$$

In order to keep the average value of the image add 1 do the centre element of the Laplace mask

Other high-pass filters

$$h'_3 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$h'_4 = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

High-pass filters

Blurred image

Sharpened image

```
%MATLAB
out_image = filter2(filter_mask, in_image);
```

Nonlinear filters

The filtered image is defined by a non-linear function of the source image

Can we compute spectral characteristics for nonlinear filters?

NO

Because transfer characteristics of nonlinear filters depend on image content itself!

Median filter (order statistic filter)

The median $\boldsymbol{m}$ of a set of values (e.g. image pixels in the filtering mask) is such that half the elements in the set are less than $\boldsymbol{m}$ and other half are grater than $\boldsymbol{m}$.

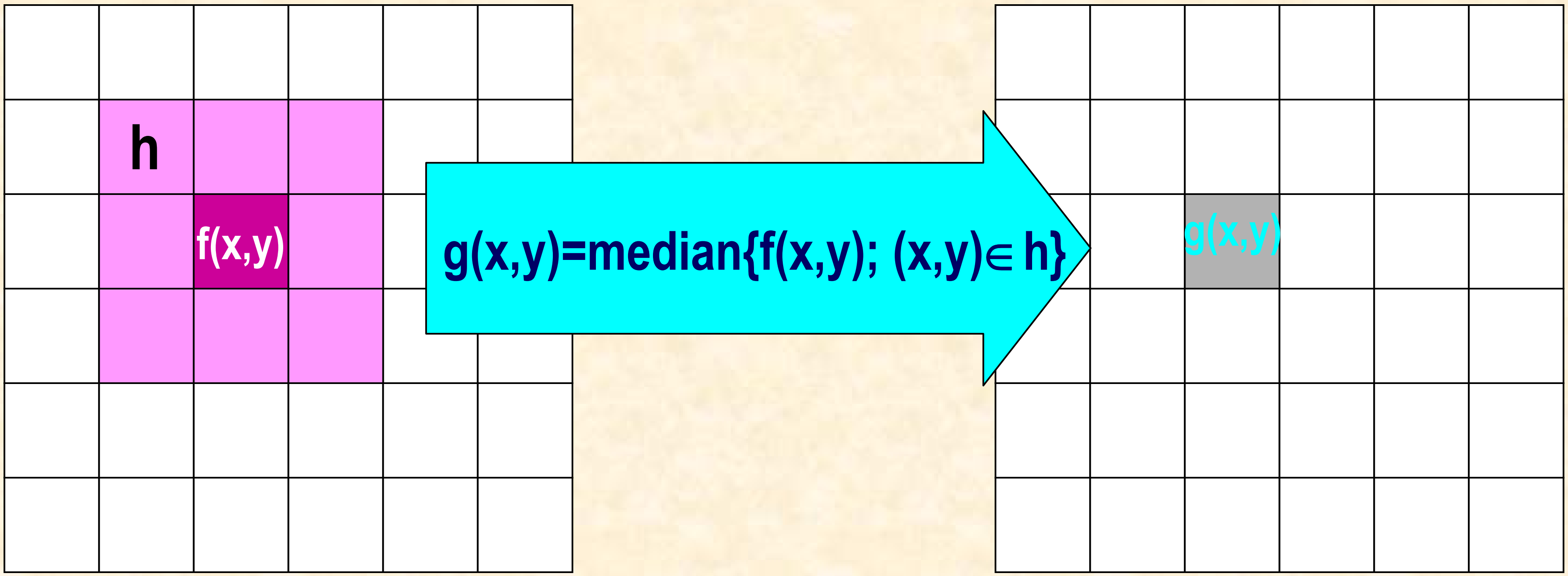
$$x(n)=\{1, 5, -7, 101, -25, 3, 0, 11, 7\}$$

Sorted sequence of elements:

$$x_s(n)=\{-25, -7, 0, 1, \mathbf{3}, 5, 7, 11, 101\}$$

median

Median filtering the image

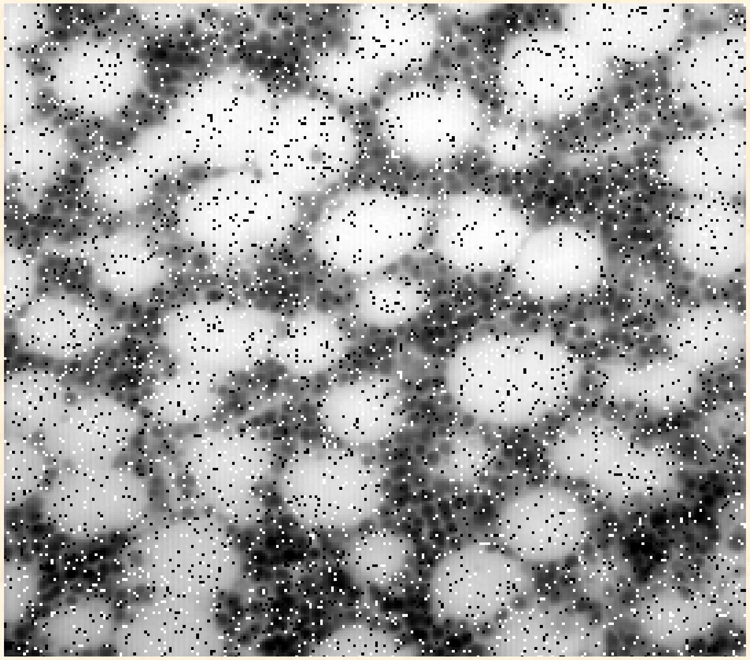


source image f

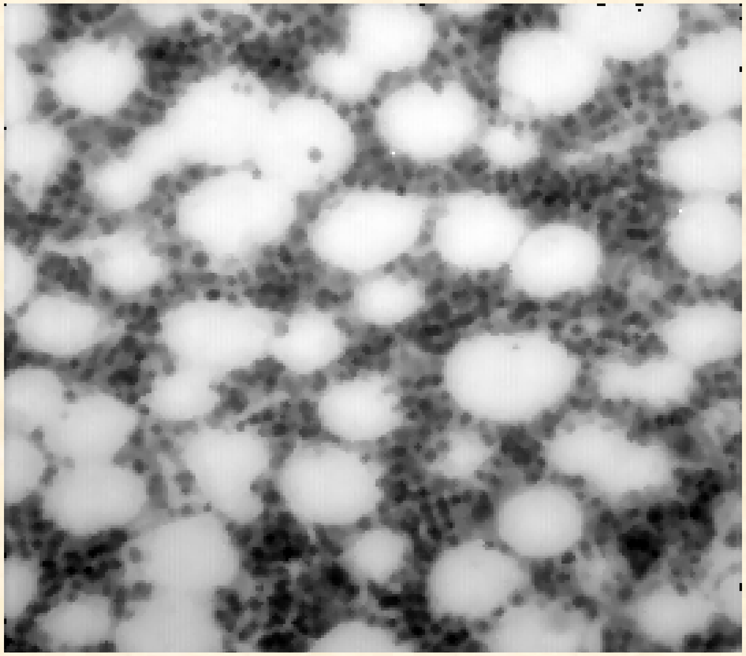
output image g

The most time consuming ioperation is sorting

Demo – median filter



Source image distorted by „salt and pepper noise”



Enhanced image using the median filter (3x3)”

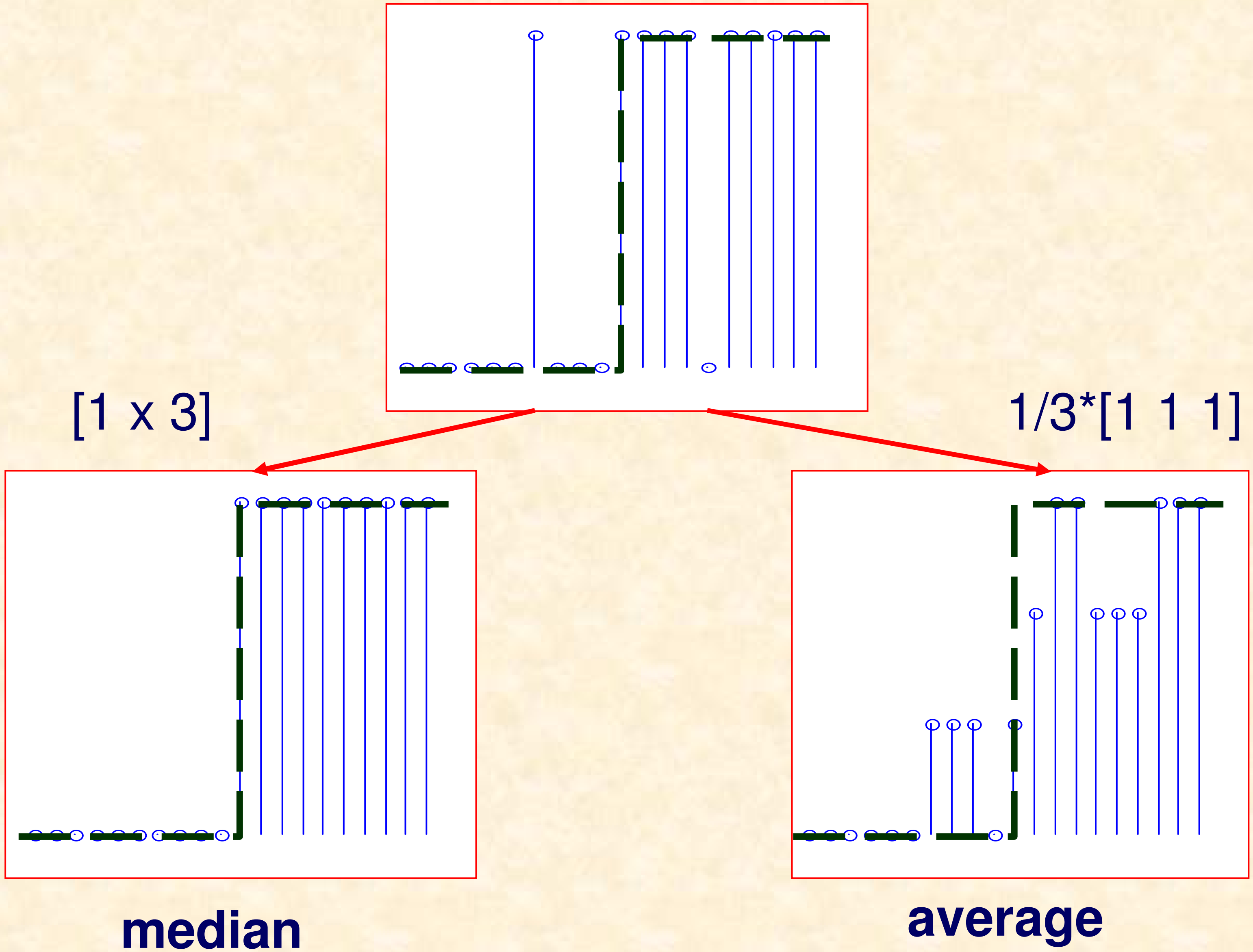
```
%MATLAB
out_image = medfilt2(in_image, [m n]);
```

Median filter

Median filter:

1. Excellent in reducing impulsive noise (od size smaller than half size of the filtering mask)
2. Keeps sharpness of image edges (as opposed to linear smoothing filters)
3. Values of the output image are equal or smaller than the values of the input image (no rescaling)
4. Large computing cost involved

Median filter



MATLAB Demo – median filter